

# Estimating Firm-Level Risk

## *Preliminary*

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### Abstract

Most models incorporating firm heterogeneity assume that idiosyncratic productivity follows an AR(1) process. However, there remains substantial disagreement on the persistence parameter, with estimates ranging from a unit root to a nearly iid component. This paper uses the information embedded in firms' investment decisions to estimate the productivity process: investment reacts more to a permanent shock than to a transitory shock. I apply this methodology to a sample drawn from Compustat, using a method of moments estimator. The estimates give an important role to permanent shocks. I study the implications of these estimates in a general equilibrium model of firm dynamics. Mistaking permanent shocks for persistent shocks can lead to incorrect inferences regarding, for instance, the effect of a friction on aggregate productivity. As an example of application of this methodology, I also study the trends and cycles in firm-level volatility.

## 1 Introduction

Panel data on firms and establishments reveal substantial idiosyncratic volatility.<sup>1</sup> This empirical finding has led to the development of models of firm dynamics with productivity heterogeneity. These models are now common in macroeconomics, industrial organization, trade, corporate finance, and other fields.<sup>2</sup> Two key inputs in these models are the variance and persistence of idiosyncratic shocks. These parameters determine the risk that firms face, their optimal policies for a given structure of adjustment costs,

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<sup>1</sup>Some key facts are: (a) sales, employment and investment are highly volatile; (b) there is a lot of turnover of jobs and firms; (c) productivity heterogeneity is large and persistent. As a result, at a point in time in a given industry, gross entry and exit are both large, even if net entry is small at the industry-level. Similarly, gross job creation and gross job destruction are large, even if net employment growth is small (see Dunne, Roberts and Samuelson (1989), Davis, Haltiwanger and Schuh (1996), and Bartelman and Doms (2000)).

<sup>2</sup>A list which makes no attempt at exhaustivity includes (1) models of factor adjustment costs (e.g., Caballero and Engel (1999) or Cooper and Haltiwanger (2006)); (2) models of entry, exit, and reallocation, (e.g., Hopenhayn (1992) or Hopenhayn and Rogerson (1993)); (3) trade theory (Melitz (2003)); (4) corporate finance (e.g. Gomes (2001), Hennessy and Whited (2005)); (5) public finance (e.g., Gourio and Miao (2008)). While most of these models have been used to study steady-states or balanced growth, recent work incorporates business cycles dynamics.

and the benefits of reallocation of inputs across firms. At the macro level these parameters affect total factor productivity, and business cycle dynamics.

However, despite the importance of these parameters, there is still considerable uncertainty surrounding them, especially the persistence of shocks. For instance, Cooper and Ejarque (2003) or Gilchrist and Sim (2007) estimate a serial correlation around 0.1-0.3, while Caballero and Engel (1999) or Bloom (2007) assume unit roots. Cooper and Haltiwanger (2006), Gomes (2001) or Hennessy and Whited (2005) fall inbetween, with a serial correlation in the 0.7-0.9 range. Given the interest in models of industry dynamics, it seems important to obtain more precise estimates of this important parameter.

Moreover, some facts suggest that the standard AR(1) modeling device, while convenient and realistic, does not capture the entire story. If shocks truly were stationary, we would see firms' sales (or employment) oscillating around a fixed size. There is considerable anecdotal and suggestive evidence that firms are also subject to permanent shocks. This motivates me to introduce permanent shock in my empirical framework.

The key idea of the paper is to introduce a novel procedure to estimate the risk faced by firms. The simple insight is that a firm invests more when it expects its productivity to be high in the future. Hence, investment data *reveal* the firm's expectations and are thus informative on the persistence of productivity. I use a simple structural adjustment cost model of investment, and I allow for a fairly general stochastic process for shocks, that includes permanent as well as transitory components. The adjustment cost model shows how to use *jointly* data on productivity (profitability) and investment to infer the dynamic properties of productivity. This procedure leads to a decomposition of productivity shocks into a permanent shock, a transitory shock and an iid shock (i.e., measurement error). By using more data and imposing more economic structure, this procedure should lead to more precise estimates, while taking into account the important measurement error. This is a substantial progress over univariate decompositions based on productivity *alone*.

This insight is implemented through a simulated method of moment estimator.<sup>3</sup> The mapping between some second moments of the data (mostly investment rates and profit rates) and the parameters describing the shock process is relatively transparent. The estimation procedure is run on data drawn from Compustat. Overall, the estimates suggest that permanent shocks are important, with the standard deviation of the innovation estimated to be about .20.<sup>4</sup>

A natural concern is that the adjustment cost model is the wrong starting point, because of lumpiness at the establishment level. However, our data is based on large firms, for which investment spikes are less apparent. As argued by Eberly, Rebelo and Vincent (2008), the adjustment cost model is a good, parsimonious model to start with, once we allow for a more general shock process.<sup>5</sup> Moreover, the basic insight that a firm invests more if it expects a more persistent shock is likely to be robust. (Finally, it is easy to assess how much an alternative model would affect the estimation procedure, by simulating data from the alternative model and running the estimation procedure on these simulated data.)

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<sup>3</sup>It is also possible to use maximum likelihood. This is work in progress.

<sup>4</sup>This number is for the shock to the profit function (which is larger than the shock to the production function by a factor of about three).

<sup>5</sup>Gilchrist and Himmelberg (1995) also find that the cross-equation restrictions implied by the (constant return) adjustment cost model are not rejected for the large and/or financially unconstrained firms.

As an example of application, the procedure is used to measure the changes in the process of idiosyncratic shocks since 1965 in the United States. Did idiosyncratic firm-level risk increase? Is idiosyncratic risk countercyclical? There is an ongoing debate regarding these views. Comin and Philippon (2005) and Davis, Haltiwanger, Jarmin and Miranda (2006) debate whether idiosyncratic risk has increased or decreased in the United States. Eisfeld and Rampini (2006) and Bloom (2008) document that firm-level idiosyncratic risk is usually countercyclical.

### **Organization of the paper**

Section 2 reviews the related literature and discusses two simple examples which illustrate the importance of the persistence of shocks. Section 3 presents some simple data suggesting a role for permanent shocks. Section 4 presents the model, estimation procedure and tests the procedure by Monte-Carlo simulations. Section 5 presents the data and results from the estimation. Section 6 studies the implications of the estimates in a general equilibrium model of firm dynamics. Section 7, as an application of the methodology, studies the time variation in idiosyncratic volatility.

## **2 Literature Review**

Productivity heterogeneity has been emphasized in the recent industrial organization and in the trade literature (e.g., Syverson (2004), Melitz (2003)). In the typical four-digit industry, the ratio of the labor productivity of the 25th centile producer to the 75th centile producer is about 2. The ratio of the labor productivity of the 90th centile producer to the 10th centile producer is about 4. Using total factor productivity (TFP) rather than labor productivity, the productivity differentials are somewhat smaller, but still large, respectively 1.4 and 2.<sup>6</sup> Controlling for observables such as vintage or capital intensity does not explain a large share of productivity heterogeneity.

While this heterogeneity is widely accepted, the interpretation in terms of variance and persistence of shocks is more controversial, as noted in the introduction. Many researchers fit models with firms fixed effects, which leads to estimates of relatively low persistence, while some researchers assume a unit root process. The evidence based on univariate decompositions of measured productivity is problematic since measurement error, which is likely to be important in these data, biases the estimates of persistence down. Moreover, productivity is rarely measured directly: it must be inferred as the residual from a production function estimation, which faces the usual endogeneity problem. Some of the estimates are based on Simulated Method of Moments estimations, which are model-dependent. More precisely, the source of identification of the shock variances is sometimes not transparent, since the estimates often try to fit better some other moments of interest. While my procedure suffers from the same limitation, the structure is minimal (adjustment costs and decreasing returns) and the relation between data moments and shocks is more transparent.

There is an important related IO literature, starting from the seminal contribution of Oley and Pakes (1996). These authors show how to estimate productivity from input choices, when the productivity process follows a univariate process. Their method does not apply to my framework since I emphasize multivariate processes, and it is thus not possible to invert the productivity shock from the input choice

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<sup>6</sup>These numbers are drawn from Syverson (2004), Table 1.

alone. On the other hand, my estimation method relies on functional form assumptions or a linear approximation, while their procedure is nonparametric. My objectives are more limited than Oley and Pakes, in that I do not wish to measure productivity *for each firm*: I am only after the parameters of the general productivity process.

There is a strong analogy between my proposed methodology and the decomposition of income into permanent and transitory shocks proposed by Blundell and Preston (1998). Blundell and Preston use the permanent income (PIH) model to study consumption and income volatility. Under the PIH, consumption is primarily determined by permanent income, and hence measuring consumption volatility allows to measure the volatility of the shock to permanent income, while the volatility of income is a mix of the volatility of permanent and transitory shocks. My paper is perhaps most closely related to Abbring and Campbell (2006) who employ a similar insight to study firms' exit and learning. Their sample (Texas bars), while adapted to their question, is somewhat less interesting from a macroeconomic perspective than Compustat.

Risk is sometimes measured using stock returns. While stock returns are likely to be a precise measure of risk, they can offer no guidance regarding the persistence of shocks: stock returns are roughly iid, whatever the persistence of the productivity (profitability) process. Moreover, in my model, marginal  $q$  differs from average  $q$ , hence it is not appropriate to measure Tobin's  $q$  from stock market data. Hence incorporating Tobin's  $q$  data in the estimation is unattractive.

More generally, the importance of disentangling permanent and transitory income shocks is of course a classical theme of modern macroeconomics. For instance, Quah (1989) shows that the presence of multiple shocks can rationalize the excess sensitivity puzzle. Similarly, the investment and corporate finance literature has recently stressed the role of the persistence of productivity; for instance, the fact that investment is sensitive to cash flow is often rationalized by the persistence of cash flow shocks. The presence of two shocks allows, in principle, to disentangle the pure effect of a cash flow increment and an increase in expected future profitability.

The persistence parameter also plays a key role in studies of reallocation. For instance, in Gourio and Miao (2008), the productivity and welfare effects of a dividend tax cut depend heavily on this parameter. Another reason why the persistence parameter is important is that it is required to estimate adjustment costs. Many researchers estimating adjustment costs model proceed in two steps, by first measuring and fitting a process to productivity, then estimating the technological or financial frictions (e.g., Cooper and Haltiwanger (2007), Fuentes, Gilchrist and Rysman (2006), or Gilchrist and Sim (2007)). To the extent that the process fitted to productivity does not fully capture the risks faced by firms, this would affect their estimates of adjustment costs.

## 2.1 The role of the persistence of shocks: two simple examples

This section illustrates the importance of the persistence of idiosyncratic shocks to productivity by studying two simple examples where pencil-and-paper results are available.

### 2.1.1 Reallocation and the persistence of shocks

Consider the following economy. There is a measure one of firms, each of which operates the production function  $f(z, k) = zk^\alpha$ , where  $z$  is productivity and  $k$  is capital. There is no entry or exit. Firm-level productivity evolves according to the following first-order process:

$$\log z_{t+1} = \rho \log z_t + \sigma \varepsilon_{t+1}. \quad (1)$$

The shock  $\varepsilon_t$  is independent across firms and time and normally distributed with unit variance. The economy is in a stationary equilibrium, so that aggregates are constant.<sup>7</sup> Assume that there are no adjustment costs, but that capital must be chosen one period in advance. The firm's objective at time  $t$  is to maximize by choice of investment  $\{i_{t+j}\}_{j=0}^\infty$ :

$$\begin{aligned} E_t \sum_{j=0}^{\infty} \beta^j (z_{t+j} k_{t+j}^\alpha - i_{t+j}), \\ \text{s.t.} \quad : \quad k_{t+j+1} = (1 - \delta)k_{t+j} + i_{t+j}. \end{aligned}$$

given an initial condition for  $k_t$ . Taking first-order conditions yields:

$$\alpha E_t (z_{t+1}) k_{t+1}^{\alpha-1} = \frac{1}{\beta} - 1 + \delta,$$

i.e. the optimal decision equates the expected marginal product of capital (the left-hand side) and the user cost of capital (the right-hand side). This can be rewritten as

$$\log k_{t+1} = \frac{1}{1 - \alpha} \log E_t z_{t+1} + \text{constant}, \quad (2)$$

where the constant in this equation (and in all equations below) does not depend on  $\rho$ . Given (1), we have  $\log E_t z_{t+1} = \rho \log z_t + \frac{\sigma^2}{2}$ , hence:

$$\log k_{t+1} = \frac{\rho}{1 - \alpha} \log z_t + \text{constant}.$$

It follows that investment is

$$\begin{aligned} i_t &= k_{t+1} - (1 - \delta)k_t \\ &\simeq \text{constant} \times \frac{\rho}{1 - \alpha} [\log z_t - (1 - \delta) \log z_{t-1}], \end{aligned}$$

so that the response of investment to a one-standard deviation shock  $\varepsilon_t$  is

$$\frac{\partial i_t}{\partial \varepsilon_t} \simeq \text{constant} \times \frac{\rho}{1 - \alpha}.$$

The firm invests more in response to a more persistent shock, because it expects the productivity to be still high in the future.

Moreover, the size of  $\rho$  affects reallocation and thus aggregate TFP. Total output in this economy is

$$Y = \int \int z k^\alpha dF(k, z),$$

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<sup>7</sup>It is straightforward to embed this model in a general equilibrium model, as in Gomes (2001) or Hopenhayn and Rogerson (1993).

where  $F$  is the joint cross-sectional distribution of  $(k, z)$ . Simple computations show that

$$\log Y = \frac{\rho^2}{2(1-\alpha)^2} \text{Var}(\log z) + \text{constant}.$$

Moreover, if one were to compute aggregate TFP as the Solow residual in this economy

$$TFP = \frac{Y}{K^\alpha},$$

the result would be

$$\log TFP = \frac{\rho^2}{2(1-\alpha)} \text{Var}(\log z) + \text{constant}.$$

Hence, for a given cross-sectional variance of log productivity  $\text{Var}(\log z)$ , aggregate output and aggregate TFP are both increasing in the persistence of the shock  $\rho$ . Given the one-period time-to-build, firms are better able to forecast their future capital needs when shocks are more persistent. Hence, the allocation of capital is more efficient when shocks are more persistent. Many researchers study the effect of impediments to reallocation such as labor market frictions, adjustment costs, or taxes (e.g. Atkeson, Khan and Ohanian (1993), Gourio and Miao (2008), Hopenhayn and Rogerson (1993)). In general in these models, the aggregate effects of these impediments to reallocation are highly sensitive to the persistence of shocks.

### 2.1.2 Irreversibility and the persistence of shocks

As an alternative example of the importance of the persistence of shocks, consider the following model of firm investment subject to irreversibility. The firm maximizes its expected discounted profit, subject to the constraint that investment must be nonnegative:

$$\begin{aligned} V(k_0, z_0) &= E_t \sum_{j=0}^{\infty} \beta^j (z_{t+j} k_{t+j}^\alpha - i_{t+j}), \\ \text{s.t.} \quad & i_{t+j} = k_{t+j+1} - (1-\delta)k_{t+j} \geq 0. \end{aligned}$$

The following simple result illustrates that when the shock has a persistence which is low enough, irreversibility does not matter.

**Proposition 1** (1) Assume that  $\{z_t\}$  is iid, and let  $k^*$  satisfy the equation:  $E(z)\alpha k^{*\alpha-1} = \frac{1}{\beta} - 1 + \delta$ . Then, if  $k_0 < \frac{k^*}{1-\delta}$ , the path  $k_t = k^*$  for  $t = 1, 2, \dots$  is optimal. Hence, we have exactly the same outcomes as in the model without irreversibility, regardless of the realization of the shocks. (2) Assume that  $\{\log z_t\}$  follows an AR(1) process, so that  $\log z_{t+1} = \rho \log z_t + u_{t+1}$ , with  $u_t$  iid. Then, if  $\rho > 0$ , if  $\delta < 1$ , and if the lowest possible value for  $u$  is small enough, the irreversibility affects the equilibrium path.

**Proof.** (1) is obtained by simply checking that the conjectured sequence satisfies the first order conditions. For (2), compute the sequence  $(k_t)$  that solves the problem under no irreversibility, and note that if  $u$  is low enough it may not be feasible. ■

This result illustrates that a necessary condition for irreversibility to matter is that  $\rho > 0$ , i.e. productivity must be forecastable. Given a positive  $\rho > 0$ , irreversibility may matter if  $\delta$  is low enough and the support of  $u$  large enough. Clearly, the larger the  $\rho$ , the “more likely” it is that irreversibility will affect the equilibrium path.

### 3 Suggestive Evidence for Permanent Shocks in Firm Dynamics

This section discusses some simple facts that motivate the introduction of permanent shocks. Figure 1 presents the path of sales of four firms drawn from a balanced sample of Compustat (1980-2006), which is described in more details in Section 5. Of course, there is a wide variety of experiences amongst firms. It is striking, though, that many firms are growing, and sometimes shrinking, without clear mean reversion. In contrast, Figure 2 presents the path of sales of four firms simulated from a standard industry model (similar to Gomes (2001) or Gourio and Miao (2008)): firms are subject to AR(1) shocks, with persistence .72, and they accumulate capital in response to these shocks, subject to adjustment costs. In this second figure, the mean-reversion is more apparent.

An alternative way to make this simple remark is to consider the correlation of the sales at time  $t$  and at time  $t+k$  for a panel of firms. In the data, this correlation decreases almost linearly with  $k$ , as shown in Figure 3. However, in a standard industry model, this correlation converges to a finite number after a certain number of years, as illustrated in Figure 4. This finite number is the share of fixed heterogeneity. This share is often assumed to be zero in the models. (If it is zero, the model predicts counterfactually that sales today and sales fifteen years from now are not correlated at all; hence I incorporate fixed heterogeneity in productivity, corresponding to the fixed effects that are used in the estimation. I pick the size of this fixed heterogeneity to match the heterogeneity in the data.) Thus as time elapses, the model imply very different behavior: in the model, firms oscillate around a fixed size, while in the data, they appear to drift one way or another. Note that these conclusions are drawn from a balanced sample, which likely underestimates the importance of permanent shocks, which may lead to entry or exit.

These patterns should not be surprising: many large firms that exist today, such as Wal-Mart, Dell, Google, etc., either did not exist, or were very small twenty years ago. Comin and Philippon (2005) also document the large amount of turnover in industry leaders. Stationarity test reject the null hypothesis of no unit root, as shown by Franco and Philippon (2007) who also document the importance of permanent shocks at the firm level using a VAR methodology. Permanent shocks lead to a ‘view of the world’ that is quite different from standard industry models without permanent shocks. From an economic point of view, the question is, do new firms matter for long-term productivity? If yes, incorporating permanent shocks appears important.

It is also interesting that a separate literature, which tries to match the firm size distribution, has emphasized the importance of permanent shocks (e.g. Luttmer 2007, Gabaix 1999; see also Miao (2005)).<sup>8</sup>

[To add: nonparametric estimates of permanent shocks a la Cochrane (1988); but short sample]

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<sup>8</sup>At a fundamental level, this is based on the fact that a geometric brownian motion with a lower barrier yields a Pareto stationary distribution, as emphasized by Gabaix (1999).

## 4 Model and Estimation Method

Before turning to the full model and estimation method, I illustrate the key idea of the procedure in a simple setup.

### 4.1 Methodology: a simple example

Consider the baseline quadratic adjustment cost model, with constant returns in profits and in adjustment costs. Hayashi's results imply that average  $q$  equals marginal  $q$ , and that investment is an affine (linear) function of average  $q$ .

The profit function is linear, i.e.  $\Pi_{it} = A_{it}K_{it}$  where  $A_{it}$  is a measure of profitability (or productivity),  $i$  indexes firms, and  $t$  indexes time. Assume that profitability is the sum of a persistent component and an iid component:

$$\frac{\Pi_{it}}{K_{it}} = A_{it} = \text{constant} + \underbrace{z_{it}}_{AR(1)} + \underbrace{\varepsilon_{it}^M}_{IID}, \quad (3)$$

$$z_{it} = \rho z_{it-1} + \eta_{it}.$$

The first-order condition for investment yields:

$$\frac{I_{it}}{K_{it}} = \text{constant} + \eta q_{it}, \quad (4)$$

where  $\eta$  is the adjustment cost parameter, and  $q_{it}$  is marginal  $q$ , the expected present discounted value of the marginal product of capital, which is approximately:<sup>9</sup>

$$q_{it} \simeq E_t \sum_{k \geq 1} \beta^k (1 - \delta)^{k-1} z_{it+k} = \frac{\beta \rho z_{it}}{1 - \beta(1 - \delta)\rho}, \quad (5)$$

where  $\rho$  is the autocorrelation of  $z_{it}$ . Putting equations 3, 4 and 5 together, we obtain the following moments:<sup>10</sup>

$$\begin{aligned} \text{Var} \left( \frac{\Pi_{it}}{K_{it}} \right) &= \sigma_z^2 + \sigma_\varepsilon^2, \\ \text{Var} \left( \frac{I_{it}}{K_{it}} \right) &= \frac{\beta^2 \rho^2 \eta^2 \sigma_z^2}{(1 - \beta(1 - \delta)\rho)^2}, \\ \text{Cov} \left( \frac{I_{it}}{K_{it}}, \frac{\Pi_{it}}{K_{it}} \right) &= \frac{\beta \rho \eta \sigma_z^2}{1 - \beta(1 - \delta)\rho}. \end{aligned}$$

Assume for simplicity that we know the structural parameters  $\beta, \delta$  and  $\eta$ , but that we ignore the parameters describing the shock processes  $\sigma_z^2, \sigma_\varepsilon^2$  and  $\rho$ . Then, the three moments allow us to identify these three parameters: we obtain  $\rho$  from the slope in a regression of profit rate on investment:

$$\frac{\text{Cov} \left( \frac{I_{it}}{K_{it}}, \frac{\Pi_{it}}{K_{it}} \right)}{\text{Var} \left( \frac{I_{it}}{K_{it}} \right)} = \frac{1 - \beta(1 - \delta)\rho}{\beta \eta \rho},$$

<sup>9</sup>The formula below neglects the reduction in future adjustment costs due to investment today.

<sup>10</sup>These moments are both cross-sectional moments and time-series moments, since we work under the assumption that there are no fixed effects.



which intuitively corresponds to the idea that a high  $\rho$  implies a high response of investment to an increase in profits. Next, we use the variance of investment rates  $Var\left(\frac{I_{it}}{K_{it}}\right)$  to determine  $\sigma_z^2$ . Finally we obtain  $\sigma_\varepsilon^2$  from the variance of profit rates  $Var\left(\frac{\Pi_{it}}{K_{it}}\right)$ . The identification is driven by the implication of the adjustment cost model that investment does not respond to an iid shock.

This relatively transparent mapping from data moments to parameter estimates is appealing, because it is clear which features of the data drive the estimates. Of course, if the model is wrong, the estimates might be highly misleading. However, just like Blundell and Preston (1998) use the permanent income model, even though it does not fit the data perfectly, it seems reasonable to use the adjustment cost model, which provides a decent description of the data, as argued by Eberly, Rebelo and Vincent (2008).

## 4.2 Model

I now use a standard partial-equilibrium, adjustment cost model that is richer than the example above: it allows for decreasing return to scale, permanent shocks to productivity, and for shocks to adjustment costs.

Assume the profit function is  $\pi(z, K) = zK^\alpha$  where  $z$  is productivity (profitability) and  $K$  is the current capital stock. This profit function can be derived as the maximized value of profits, when variable factors have been optimized out. As usual, the exponent  $\alpha$  reflects decreasing return to scale or curvature in demand.<sup>11</sup>

The firm accumulates capital subject to adjustment costs:

$$C(I, K, \varepsilon^{AC}) = K \times c\left(\frac{I}{K} - \varepsilon^{AC}\right),$$

where  $c$  is a convex function satisfying  $c(0) = 0$ ,  $c'(x) > 0$  for  $x > 0$  and  $c'(x) < 0$  for  $x < 0$ .  $\varepsilon^{AC}$  is a shock to the adjustment cost, which is assumed to be *iid* across firms and across time, and normally distributed with variance  $\sigma_{AC}^2$ . Narrowly, the shock  $\varepsilon^{AC}$  captures changes in the cost of adjustments, and more broadly it captures deviation from the smooth adjustment cost model. However, note that this formulation of adjustment costs does not encompass the foregone profits emphasized by Cooper and Haltiwanger (2006) among others.<sup>12</sup>

Productivity shocks are of course likely the most important.<sup>13</sup> I will assume the following decomposition of profitability  $z$  into permanent and transitory components:

$$\log z_t = \log z_t^P + \log z_t^T + \varepsilon_t^M,$$

with

$$\log z_t^P = \log z_{t-1}^P + \varepsilon_t^P,$$

and

$$\log z_t^T = \rho \log z_{t-1}^T + \varepsilon_t^T,$$

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<sup>11</sup>Deviating from constant returns is important because the constant return model does not fit the micro data well (i.e. marginal  $q$  is not equal to average  $q$ ) and, most importantly, because it allows to have a non-stationary process for  $z$  while keeping the profit rate stationary.

<sup>12</sup>i.e., the idea that the adjustment cost is proportional to current profits due to a disruption in the production process.

[Further work will investigate the robustness of the estimation to this kind of model deviations.]

<sup>13</sup>Note that these could be “demand” shocks as well as “supply shocks” (Foster, Haltiwanger, and Syverson (2008)).

with  $\rho < 1$ . Here,  $\varepsilon_t^M, \varepsilon_t^T$ , and  $\varepsilon_t^P$  are all *iid* across firms and time, and normally distributed with variances  $\sigma_M^2, \sigma_T^2$ , and  $\sigma_P^2$ . These shocks are further independent of each other.  $\varepsilon^M$  can be interpreted as measurement error in profits (or a profit windfall). The two other shocks determine the importance of permanent shocks and of transitory shocks. The only difference between the two is the long-run effect: a permanent shock will lead to a permanent increase in the capital stock and hence, through capital deepening, a further increase in sales and profits. In contrast, a transitory shock has no long-run effect and the firm returns after a while to its initial size.

The firm's problem is to maximize its value by choice of investment:

$$\begin{aligned} V(K_0, z_0^P, z_0^T) &= \max_{\{I_t, K_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left( z_t K_t^\alpha - I_t - K_t c \left( \frac{I_t}{K_t} - \varepsilon_t^{AC} \right) \right), \\ \text{s.t.} \quad &: K_{t+1} = (1 - \delta)K_t + I_t, \\ &K_0 \text{ given, law of motions for shocks.} \end{aligned} \quad (6)$$

The Bellman equation is:

$$\begin{aligned} V(K, z_p, z_t, \varepsilon_{ac}, \varepsilon_m) &= \max_I \left\{ \begin{aligned} &z_p z_t e^{\varepsilon_m} K^\alpha - I - K c \left( \frac{I}{K} - \varepsilon^{AC} \right) \\ &+ \beta E_{\varepsilon'_p, \varepsilon'_t, \varepsilon'_{ac}, \varepsilon'_m} V(K', z'_p, z'_t, \varepsilon'_{ac}, \varepsilon'_m) \end{aligned} \right\} \\ \text{s.t.} \quad &: K' = (1 - \delta)K + I, \\ &z'_p = z_p e^{\varepsilon'_p} \text{ and } z'_t = z_t^\rho e^{\varepsilon'_t}. \end{aligned}$$

However, given that both  $\varepsilon_m$  and  $\varepsilon_{ac}$  are *iid*, this can be rewritten in a simpler way as:

$$\begin{aligned} V(K, z_p, z_t) &= E_{\varepsilon_m} (z_p, z_t e^{\varepsilon_m} K^\alpha) + E_{\varepsilon_{ac}} \max_I \left\{ -I - K c \left( \frac{I}{K} - \varepsilon_{ac} \right) + \beta E_{\varepsilon'_p, \varepsilon'_t} V(K', z'_p, z'_t) \right\} \\ \text{s.t.} \quad &: K' = (1 - \delta)K + I, \\ &z'_p = z_p e^{\varepsilon'_p} \text{ and } z'_t = z_t^\rho e^{\varepsilon'_t}. \end{aligned}$$

This can be further simplified given the assumption that  $z_p$  is permanent, because  $V$  is homogeneous in  $(z^{1-\alpha}, K)$ . The solution to the Bellman equation leads to the following policy functions:

$$\begin{aligned} I &= I(K, z_p, z_t, \varepsilon_{ac}), \\ K' &= K'(K, z_p, z_t, \varepsilon_{ac}), \\ \pi &= \pi(K, z_p, z_t, \varepsilon_m). \end{aligned}$$

Note how profits are affected by  $\varepsilon_m$  but not by  $\varepsilon_{ac}$ , and inversely investment is affected by  $\varepsilon_{ac}$  but not by  $\varepsilon_m$ . This is the source of identification of these shocks. Taking first-order condition in (6) yields the standard  $q$ -theory result:

$$q_t = 1 + c' \left( \frac{I_t}{K_t} - \varepsilon_{ac} \right), \quad (7)$$

where  $q_t$  is marginal  $q$ , i.e. the multiplier on the capital accumulation constraint. Moreover,

$$\begin{aligned} q_t &= \beta(1 - \delta)E_t(q_{t+1}) + \beta\alpha E_t(z_{t+1}K_{t+1}^{\alpha-1}) \\ &\quad + \beta E_t \left( -c(I_{t+1}/K_{t+1} - \varepsilon_{t+1}^{AC}) + I_{t+1}/K_{t+1} c'(I_{t+1}/K_{t+1} - \varepsilon_{t+1}^{AC}) \right), \end{aligned} \quad (8)$$

which shows that marginal  $q$  is related to the present discounted value of marginal product of capital, plus terms reflecting changes in adjustment costs. As an illustration, when the adjustment costs terms can be neglected or cancel out, we have

$$q_t = \alpha E_t \sum_{j=1}^{\infty} \beta^j (1 - \delta)^j z_{t+j} K_{t+j}^{\alpha-1}, \quad (9)$$

which clearly shows that marginal  $q$  reflects the firm's long-term expectation of its marginal product of capital. Marginal  $q$  is unobservable, but the first-order condition (7) shows that investment is related to marginal  $q$ .

### 4.3 Solution method and Impulse Responses

The model has no closed-form solutions, hence we need to solve it numerically. Because the model has many state variables and shocks, and must be solved repeatedly for the estimation, it seems reasonable as a first approach to use a linear solution method. First, make the model stationary by constructing 'trend-adjusted' variables  $k_t = K_t/z_{P,t-1}^{\frac{1}{1-\alpha}}$ , and  $i_t = I_t/z_{P,t-1}^{\frac{1}{1-\alpha}}$ . Next, rewrite the first-order conditions using these detrended variables, and find the associated nonstochastic steady-state (i.e. set  $\varepsilon^T = \varepsilon^M = \varepsilon^{AC} = 0$ ). Finally, write a linear approximation to the first-order conditions around the steady-state. This yields a linear rational expectations model which can be easily solved by the method of undetermined coefficients. More precisely, the policy rule are:

$$\begin{aligned} \widehat{k}_{t+1} &= a_0 \widehat{k}_t + a_1 \log z_t^T + a_2 \varepsilon_t^{AC} + a_3 \varepsilon_t^P, \\ \widehat{i}_t &= e_0 \widehat{k}_t + e_1 \log z_t^T + e_2 \varepsilon_t^{AC} + e_3 \varepsilon_t^P, \\ \widehat{q}_t &= b_0 \widehat{k}_t + b_1 \log z_t^T + b_2 \varepsilon_t^{AC} + b_3 \varepsilon_t^P, \\ \widehat{\pi}_t &= \alpha \widehat{k}_t + \log z_t^T + \varepsilon_t^M + \varepsilon_t^P, \end{aligned}$$

where the coefficients  $a$ 's,  $b$ 's and  $e$ 's can all be easily found using a quadratic equation. Once the model is solved in terms of detrended variables, it is easy to find the path for the level variables  $K_t, I_t, \pi_t$ . To illustrate the workings of the model, Figure 5 shows the responses of the model to a permanent shock  $\varepsilon^P$ , a transitory shock  $\varepsilon^T$ , an adjustment cost shock  $\varepsilon^{AC}$ , and an iid shock  $\varepsilon^M$ .<sup>14</sup> In response to a permanent shock, investment increases by a large amount, and capital (size) goes up permanently. A transitory shock leads to a temporary increase in size which is reverted in the long-run. An iid shock to profit has no effect on anything but profits. Finally, an adjustment cost shock affects investment, but not today's profits. Moreover, the profit rate increases as the capital decreases (as opposed to temporary positive productivity shock).

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<sup>14</sup>For these figures, I assume that adjustment cost is quadratic:  $c(x, \varepsilon) = \frac{\psi}{2}(x - \delta - \varepsilon)^2$ , and I set the following parameter values:  $\psi = 1$ ,  $\delta = 0.20$ ,  $\alpha = 0.57$ ,  $\beta = .93$ ,  $\rho = 0.8$ . Because this is a linear solution, the shock variances do not affect the decisions of the firm (there is certainty equivalence). The linear solution also makes it relatively easy to implement a maximum likelihood estimator (see below), and is appealing since identification is driven by second-order moments, which are easier to understand.

## 4.4 Identification

The intuition for the identification is mostly straightforward. The shock  $\varepsilon_t^{AC}$  captures changes in investment that are uncorrelated with current or future profits. The shock  $\varepsilon_t^M$  captures changes in profits that are uncorrelated with current (and future) investment. The shocks  $\varepsilon_t^T$  and  $\varepsilon_t^P$  affect both profits and investment,  $\varepsilon_t^P$  however has more persistent effect and is the only one to affect the firm size even in the long run. This suggests that in order to estimate the variance of these shocks as well as  $\rho$ , it is necessary to use either a long-run measure of firm size. Given the other parameters, the adjustment cost parameter  $\eta$  is identified off the serial correlation of investment, or the correlation between investment and profits.

## 4.5 Estimation method

The first estimation method involves picking parameter to minimize the discrepancy between data moments and model moments.<sup>15</sup> The model moments are obtained by simulating the model. (In the case of the linear solution method, it is possible to solve exactly for the moments.) This is attractive, especially in the linear case because the identification is relatively transparent when the moments are well-chosen. In our case, the autocorrelation of the investment rate is affected by the adjustment cost parameter; the variance of the investment rate is affected by the variance of the permanent and transitory shocks; the variance of the profit rate is affected by all the shocks to profits; and the covariance between profit and investment determines the split between permanent/transitory shocks on the one hand, and iid shocks on the other hand. To be able to distinguish between permanent and transitory shocks, I added more moments: first, autocovariances of investment rates and profit rates, and finally the covariance between profit rate and the 3-year growth rate. This last moment behaves very differently depending on the persistence of the shock.

To summarize, the ten moments used are:

$$\begin{aligned} & Var\left(\frac{I_{it}}{K_{it}}\right), Var\left(\frac{\pi_{it}}{K_{it}}\right), Cov\left(\frac{I_{it}}{K_{it}}, \frac{\pi_{it}}{K_{it}}\right), \\ & Cov\left(\frac{I_{it}}{K_{it}}, \frac{I_{it-1}}{K_{it-1}}\right), Cov\left(\frac{I_{it}}{K_{it}}, \frac{I_{it-2}}{K_{it-2}}\right), Cov\left(\frac{I_{it}}{K_{it}}, \frac{I_{it-3}}{K_{it-3}}\right), \\ & Cov\left(\frac{\pi_{it}}{K_{it}}, \frac{\pi_{it-1}}{K_{it-1}}\right), Cov\left(\frac{\pi_{it}}{K_{it}}, \frac{\pi_{it-2}}{K_{it-2}}\right), Cov\left(\frac{\pi_{it}}{K_{it}}, \frac{\pi_{it-3}}{K_{it-3}}\right), \\ & Cov\left(\log\frac{K_{it}}{K_{it-3}}, \frac{\pi_{it-3}}{K_{it-3}}\right). \end{aligned}$$

This allows me to estimate 6 parameters  $\eta, \sigma_P, \sigma_T, \rho, \sigma_{AC}, \sigma_M$ . I do not estimate the other parameters. The parameters  $\delta$  and  $\alpha$  are picked to match the unconditional means of the investment rate and the profit rate: marginal Tobin  $q$  is one in steady-state, and this condition together with  $\alpha$  pins down the profit rate. This leads to  $\alpha = 0.57$ . The depreciation rate pins down the mean investment rate; matching the mean requires  $\delta = 0.20$ . Finally, I set  $\beta = 0.93$  *a priori*, corresponding to a 7% annual discount rate.

In all the reported results I used the identity matrix to weight the moments. The standard errors

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<sup>15</sup>See Cooper and Haltiwanger (2006) for an application of this methodology.

are computed using the SMM formula. The data standard errors were computed by GMM using the pooled data, allowing for arbitrary time series correlation within each firm.<sup>16</sup>

It seems attractive to use “long-run moments” in the estimation, e.g. the variance ratio emphasized by Cochrane (1988), to better distinguish temporary vs. permanent shocks. The short sample makes these moments hard to use, and they require to balance the panel, which is why I have so far abstracted from them.

## 4.6 Evaluation of the Estimator: Monte-Carlo Evidence

A natural concern is that this estimation method may not allow to identify all the parameters of the productivity shock process, namely  $\sigma_p, \sigma_t, \rho$ , and  $\sigma_m$ . More precisely, for  $\rho = 0$ ,  $\sigma_t$  and  $\sigma_m$  are not separately identified, and for  $\rho = 1$ ,  $\sigma_t$  and  $\sigma_p$  are not separately identified. In general, for  $\rho$  close to zero (resp. close to one), it will be very hard to distinguish the transitory shock from the i.i.d. shock (resp. the permanent shock). The standard solution, which I adopt, is to restrict  $\rho$  to an interval  $[\underline{\rho}, \bar{\rho}]$  with  $\underline{\rho} > 0$  and  $\bar{\rho} < 1$ . In practice, I set  $\underline{\rho} = .4$  and  $\bar{\rho} = .9$ .

Even then, one may be worried that identification will not be very well behaved. To assess the performance of the estimator, I use a monte-carlo method. I simulate 200 panels of artificial data from the model.<sup>17</sup> I then estimate the implied parameters. Table 1 presents the mean and standard deviation of estimates, compared to the true values. The first three rows show the performance of the estimator when  $\sigma_t = 0$ , so that productivity shocks are only iid or permanent. The next three rows study the case when  $\sigma_p = 0$ : there are only iid or AR(1) shocks. Finally, the last three rows presents the results for the general model.

Overall the estimator seems to work well. In all cases, the mean estimate is close to the truth. The standard deviation of the estimates is low for all parameters for the first two models (A and B), and for all but one of the parameters ( $\rho$ ) in case C. Clearly, identification of  $\rho$  is nontrivial.<sup>18</sup> However, the important point for us is that even though  $\rho$  is hard to estimate precisely, this does not contaminate the other estimates ( $\sigma_m, \sigma_p, \sigma_t$ ) which appear to be quite accurate.

## 5 Empirical Results

First I discuss the data moments used in the estimation, then I present the results from the SMM estimation.

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<sup>16</sup>For the unbalanced data, the standard errors are computed assuming a perfect time series correlation within each firm. This seems to be conservative.

<sup>17</sup>These are balanced panels with  $N = 503$  and  $T = 27$ .

<sup>18</sup>It may well be possible to improve the performance of the estimator by adding or selecting more the moments. Alternatively, MLE might alleviate somewhat this “weak identification” problem.

		$\eta$	$\sigma_p$	$\sigma_t$	$\rho$	$\sigma_m$	$\sigma_{ac}$
A:Permanent shocks only	Truth	3	.10	–	–	.15	.10
	Mean Est.	2.9965	.0997	–	–	.1503	.1000
	SE Est.	.0962	.0032	–	–	.0059	.0006
B: AR(1) shocks only	Truth	3	–	.15	.75	.15	.10
	Mean Est.	3.0051	–	.1515	.7414	.1485	.0999
	SE Est.	0.2207	–	.0084	.0318	.0053	.0006
C: Full model	Truth	3	.05	.15	.65	.10	.10
	Mean Est.	3.0114	.0492	.1504	.6480	.0981	.1000
	SE Est.	.1549	.0043	.0084	.0787	.0111	.0006

Table 1: Test of the estimator: Monte-Carlo simulations. For each model (A,B or C), I simulated 200 balanced panels of artificial data using the "true" parameters ( $N=503$ ,  $T=27$ ). The SMM estimator is then used to estimate the structural parameters. The table report the mean and standard deviation, across the 200 panels, of the estimates. The three models are (A) the model with no transitory shocks, (B) the model with no permanent shocks, (C) the model with both kind of shocks.

	$\sigma_I$	$\sigma_\pi$	$\rho_{I\pi}$	$\rho_{1I}$	$\rho_{2I}$	$\rho_{3I}$	$\rho_{1\pi}$	$\rho_{2\pi}$	$\rho_{3\pi}$	$\rho\left(\log\frac{K}{K_{-3}}, \frac{\pi_{-3}}{K_{-3}}\right)$
Balanced	.12	.44	.32	.56	.39	.35	.84	.73	.66	.19
Unbalanced	.19	.61	.25	.45	.23	.11	.72	.56	.48	.26

Table 2: Data moments. The table reports the standard deviation and autocorrelations of the investment rate and profit rate as well as the correlation between the profit rate and the 3 year average growth rate of capital.

## 5.1 Data

The data are drawn from Compustat. I use two datasets: first, a balanced panel of firms (1980-2006), which is more comparable to the existing literature. For these data,  $N = 503$  and  $T = 27$ . Second, I use an unbalanced panel with  $N = 24701$  and  $T = 42$  (1965-2006). I use only data for investment  $I$  (item 30), capital  $K$  (item 8), and profits  $\pi$  (item 13). The investment rate is computed as  $I/K$  and the profit rate as  $\pi/K$ . I drop outliers (i.e. firms which have in a given year a profit rate above 4 or less than  $-.5$ , or an investment rate above 1.5).

The main moments that I will be interested are reported in Table 2. The unbalanced panel has, unsurprisingly, more volatility, but the moments are otherwise similar: profitability is highly persistent, while investment is only mildly persistent. These estimates pool the time-series and cross-section data, and assume no fixed effects.

<b>Balanced</b>	$\eta$	$\sigma_p$	$\sigma_t$	$\rho$	$\sigma_m$	$\sigma_{ac}$
Permanent shocks only	4.785	.141	–	–	.435	.111
<i>se</i>	.137	.017	–	–	.012	.003
AR(1) shocks only	5.135	–	.403	.89	.216	.113
<i>se</i>	.129	–	.038	.031	.015	.004
Full model	5.114	.117	.451	.63	.00	.112
<i>se</i>	.142	.024	.017	.041	na	.004

Table 3: Estimates using the SMM estimator for the balanced panel. N=503,T=27. Three models are estimated separately: (1) permanent shocks but no AR1 shocks; (2) AR1 shocks but no permanent shocks; (3) both shocks.

<b>Unbalanced Sample</b>	$\eta$	$\sigma_p$	$\sigma_t$	$\rho$	$\sigma_m$	$\sigma_{ac}$
Permanent shocks only	2.404	.245	–	–	.754	.175
<i>se</i>	.020	.001	–	–	.003	.001
AR(1) shocks only	2.587	–	.641	.84	.464	.178
<i>se</i>	.023	–	.004	.002	.006	.001
Full model	2.583	.203	.782	.49	.00	.177
<i>se</i>	.023	.0023	.014	.020	na	.001

Table 4: Estimates using the SMM estimator for the unbalanced panel. Three models are estimated separately: (1) permanent shocks but no AR1 shocks; (2) transitory shocks but no permanent shocks; (3) both shocks.

## 5.2 Estimates based on pooled data

Tables 3 and 4 report the parameter estimates for the three variants of the model: (i) permanent shocks but no AR(1) shocks, i.e.  $\rho = \sigma_t = 0$ ; (ii) no permanent shocks, i.e  $\sigma_p = 0$ , (iii) the full model with both kind of shocks. The results are presented both for the unbalanced and for the balanced panel. The adjustment cost parameter is always estimated to be relatively low (i.e.  $\eta$  is rather high). Of course, our estimate is not directly comparable to most of the results of the literature, which either assume constant return to scale or use average  $q$  to measure Tobin’s  $q$ .

When  $\rho = \sigma_t = 0$  (first row of each table), the estimated permanent shocks are large, and the iid shock is large too.<sup>19</sup> When we estimate only an AR(1) process, the persistence is very high, close to 0.9, but if we allow for both transitory and permanent shocks, the persistence is quite low, and the iid shock becomes insignificant. The results are similar for the unbalanced sample, except that volatility is a bit larger as expected since all the variables are more volatile. (The adjustment cost appears to be larger for the unbalanced sample.) Overall, these estimates suggest that permanent shocks are important.

<sup>19</sup>Note,when interpreting this table, that the estimated  $z$  is the true  $z$  elevated to the power  $\frac{1}{1-\nu}$ : profits (and output) amplify the variation of productivity through labor adjustment. As a result, the intrinsic shock to productivity is  $\frac{1}{1-\nu} = 3$  times less volatile.

## 6 Implications of the new estimates in a general equilibrium model

In this section, I study a general equilibrium model with ex-ante identical firms which are ex-post heterogeneous due to productivity shocks. The model is used to show how the new estimates obtained in Section 5 affect some results of the literature. More precisely, industry equilibrium models are typically used to assess the effect of frictions on aggregate outcomes such as TFP. The model allows me to conduct the following experiment: assume that the estimates of Section 5 are the truth, but that the economist falsely fits a model without permanent shocks, and then uses his results to assess the effect of removing a friction. By how much will his answers be wrong?

### 6.1 Model setup

The partial equilibrium adjustment cost model used in the estimation is embedded here in a general equilibrium model. This model can easily be modified to accommodate different adjustment cost structures and/or financing constraints, as in Cooper and Haltiwanger (2006), Gomes (2001) or Gourio and Miao (2008). The model has a continuum a firm and a representative household. The representative household has preferences

$$E \sum_{t=0}^{\infty} \beta^t U(c_t),$$

where  $U$  is an increasing and concave function. Labor supply is inelastic, equal to  $\bar{N}$ . The model features no aggregate shocks. By a law of large numbers, idiosyncratic shocks wash out, and aggregate quantities and prices are constant over time. Hence, in a stationary steady-state, firms discount future payoffs at rate  $\beta$ .

Each firm operates a Cobb-Douglas technology with decreasing return to scale:

$$Y = zK^\alpha N^\nu,$$

where  $z$  is the idiosyncratic productivity of the firm. The firm can adjust each period labor in a frictionless market, however investment in capital goods is subject to adjustment costs. It is assumed that the productivity  $\log z$  is the sum of three components:

$$\log z = \log z_p + \log z_t + \log z_m,$$

where  $\log z_m$  is *iid* across firms and time and  $N(0, \sigma_m^2)$ ,  $\log z_t$  follows a stationary Markov process with transition function  $Q$ , and  $\log z_p$  follows a normal unit root process:

$$\log z'_p = \log z_p + \mu + \sigma_p \varepsilon'_p,$$

where a prime ( $'$ ) denotes next period value,  $\mu$  is a trend, and  $\varepsilon_p$  is *iid* across firms and time and  $N(0, \sigma_p^2)$ .



The value  $W$  of a firm with current capital  $K$ , and current productivity values  $z_p, z_t, z_m$  is given by the following Bellman equation:

$$\begin{aligned}
W(K, z_p, z_t, z_m) &= \max_{N, I} \left\{ z_m z_p z_t K^\alpha N^\nu - wN - I - \phi(I, K) + \beta(1 - \eta)EW(K', z'_p, z'_t, z'_m) + \beta\eta K \right\}, \\
s.t. \quad &: K' = (1 - \delta)K + I, \\
& z_m \text{ iid with cdf } H(\cdot); \\
& z_t \text{ Markov with transition function } Q(\cdot, \cdot); \\
z_p \text{ satisfies:} & \quad \log z'_p = \log z_p + \mu + \sigma_p \varepsilon'_p, \text{ with } \varepsilon'_p \text{ iid } N(0, 1).
\end{aligned}$$

There is an exogenous rate of exit  $\eta$ . We need to introduce exit since otherwise, in the presence of permanent shocks, the firm distribution is not well defined. It is possible to make exit endogenous, but to keep the model as simple and as close to the literature as possible, we simply assume a constant rate of exit. If the firm exits, it can sell its capital.<sup>20</sup>

The function  $\phi$  represents adjustment costs.  $\phi$  is assumed to be homogeneous of degree one in  $(I, K)$ . Again to keep the model close to the literature, we abstract here from “adjustment cost shocks”. We can compute the optimal labor demand, output supply and profit function by maximizing out labor:

$$N = N(z_m, z_p, z_t, K; w) = \left( \frac{z_m z_p z_t K^\alpha \nu}{w} \right)^{\frac{1}{1-\nu}},$$

$$Y = Y(z_m, z_p, z_t, K; w) = A_1 \times (z_m z_p z_t)^{\frac{1}{1-\nu}} \times K^{\frac{\alpha}{1-\nu}},$$

$$\Pi = \Pi(z_m, z_p, z_t, K; w) = A_2 \times (z_m z_p z_t)^{\frac{1}{1-\nu}} \times K^{\frac{\alpha}{1-\nu}},$$

where  $A_1 = \left(\frac{\nu}{w}\right)^{\frac{\nu}{1-\nu}}$  and  $A_2 = A_1(1 - \nu)$ . This is of the form used in the estimation in Section 5, up to a redefinition of  $\alpha$  and of  $z$ .

We now can simplify the Bellman equation by exploiting two standard “tricks”: first,  $z_m$  is *iid* and thus does not need to be included in the state vector; hence we can rewrite the Bellman equation as

$$\begin{aligned}
V(K, z_p, z_t; w) &= \max_I \left\{ E \left( z_m^{\frac{1}{1-\nu}} \right) (z_p z_t)^{\frac{1}{1-\nu}} K^{\frac{\alpha}{1-\nu}} A_2(w) - I - \phi(I, K) + \beta(1 - \eta)EV(K', z'_p, z'_t) + \beta\eta K \right\}, \\
s.t. \quad &: K' = (1 - \delta)K + I.
\end{aligned} \tag{10}$$

The optimal solution yields the policy functions:

$$K' = g(K, z_p, z_t; w), \tag{11}$$

$$I = h(K, z_p, z_t; w).$$

(Note that we use here implicitly the fact that  $I$  and  $K'$  do not depend on  $z_m$ .) The second trick that we use is the homogeneity of the objective function. More precisely, define  $k = \frac{K}{z_p^{\frac{1}{1-\alpha-\nu}}}$ , and the investment rate  $\iota = \frac{I}{K}$ , then it is easy to verify the following guess:

$$V(K, z_p, z_t) = z_p^{\frac{1}{1-\alpha-\nu}} j(k, z_t),$$

<sup>20</sup>This assumption is relatively innocuous.

where  $j$  satisfies the following functional equation:

$$\begin{aligned}
j(k, z_t; w) &= A_2(w)E \left( z_m^{\frac{1}{1-v}} \right) z_t^{\frac{1}{1-v}} k^{\frac{\alpha}{1-v}} \\
&\quad + \max_{\iota} \left\{ -(\iota + \phi(\iota, 1))k + \beta(1 - \eta)E \left[ e^{\frac{\mu + \sigma_p \varepsilon'_p}{1-\alpha-v}} j(k', z'_t; w) \right] + \beta \eta k \right\} \quad (12) \\
s.t. \quad &: \\
k' &= \frac{K'}{z_p^{\frac{1}{1-\alpha-v}}} = \frac{K'}{K} \frac{K}{z_p^{\frac{1}{1-\alpha-v}}} \frac{z_p^{\frac{1}{1-\alpha-v}}}{z_p^{\frac{1}{1-\alpha-v}}} = (1 - \delta + \iota) k e^{\frac{-\mu - \sigma_p \varepsilon'_p}{1-\alpha-v}}.
\end{aligned}$$

This is a substantial improvement over the previous equation because (1) the state variables are now bounded (while before  $z_p$  and  $K$  were unbounded) and (2) there are only two states instead of three. We have now a dynamic programming problem with two states which can be solved using standard numerical techniques. (See the appendix for details on the numerical solution.)

To find aggregates, we need to keep track of the measure of firms with state  $K, z_p, z_t$ . This measure satisfies the following law of motion: for any (measurable) sets  $A, B, C$ ,

$$\begin{aligned}
&\mu'(A \times B \times C) \\
&= (1 - \delta) \int \mathbf{1}_{g(K, z_p, z_t) \in A} \times \Pr(z_p \exp(\mu + \sigma_p \varepsilon'_p) \in B) \times Q(z_t, C) \times \mu(dK, dz_p, dz_t) \\
&\quad + M \times v(z_p, z_t) \mathbf{1}_{K_{in} \in A}, \quad (13)
\end{aligned}$$

where  $M$  is the mass of new entrant each period,  $g$  is the policy function from (11),  $v$  is the distribution of entrants over the two shocks  $z_p$  and  $z_t$ , and  $K_{in}$  is the initial capital of entrants.<sup>21</sup>

A stationary equilibrium is defined as a value function  $W$  and policy function  $g$ , a distribution  $\mu$ , mass of entrant  $M$ , and wage  $w$  such that:

- (1) the value function  $W$  and policy function  $g$  solve (10);
- (2) the distribution  $\mu$  is self-preserving, i.e.  $\mu' = \mu$  in equation (13);
- (3) the labor market clears:

$$\int \int \int \int n(K, z_p, z_t, z_m; w) H(dz_m) \mu(dK, dz_p, dz_t) = \bar{N};$$

- (4) the free entry condition holds:

$$\int \int W(K_{in}, z_p, z_t) dv(z_p, z_t) = c_{in},$$

where  $c_{in}$  is the entry cost (in units of goods).

As a result, the goods market clears:

$$\begin{aligned}
&C + \int \int \int \int (I(K, z_p, z_t; w) + \phi(I(K, z_p, z_t; w), K)) \mu(dK, dz_p, dz_t) + M(c_{in} + K_{in}) \\
&= \int \int \int \int Y(K, z_p, z_t, z_m; w) H(dz_m) \mu(dK, dz_p, dz_t). \quad (14)
\end{aligned}$$

To solve for the equilibrium, we follow Hopenhayn and Rogerson (1993) and Gomes (2001): first, we find a wage such that the free entry condition holds; next, we find the equilibrium number of entrants which is consistent with labor market clearing.<sup>22</sup>

<sup>21</sup>Because we will assume that the adjustment cost is a function of the  $I/K$  ratio, we cannot assume that firms start with zero initial capital.

<sup>22</sup>The goods market clearing condition defines the equilibrium consumption  $C$ .

Parameter	Meaning	Value
$\alpha$	Share of capital	.2
$v$	Share of labor	.65
$\rho$	Persistence of $z_t$	.5
$\sigma_t$	Std Dev innovation $z_t$	.15
$\sigma_p$	Std Dev innovation $z_p$	.07
$\sigma_m$	Std Dev iid shock	.15
$\psi$	Adj cost parameter	$\frac{1}{3}$
$\beta$	Discount factor	.93
$c_{in}$	entry cost	1
$\delta$	Depreciation of capital	0.20
$\eta$	Exogenous exit rate	0.05

Table 5: Calibration of the general equilibrium model.

This model is thus a relatively straightforward elaboration of Gomes (2001), Gourio and Miao (2008) or Hopenhayn and Rogerson (1993), but incorporates a much richer shock structure. An advantage of this framework is that it is easy to modify the adjustment costs, and thus to assess the robustness of the results.

## 6.2 Model Calibration and Policy Functions

First, I present briefly the model's calibration and key feature. Next, I consider the experiments of changing the adjustment cost frictions. The parameters are chosen to match the estimation results (when possible). They are summarized in Table 5. The parameters  $\alpha$  and  $v$  are picked to be consistent with (a) the standard labor share of .65 and (b) the curvature coefficient of  $\frac{\alpha}{1-v} = .57$ .<sup>23</sup> The shock processes are directly given by the estimates of the unbalanced panel.<sup>24</sup> Finally, the adjustment cost parameter is taken to be  $1/3$ , corresponding to  $\eta \approx 3$ . The entry cost is normalized to 1. The initial capital is assumed to be the nonstochastic steady-state, and firms enter with  $z_p = 1$  and  $z_t$  uniformly distributed. Note that we do not need to specify the utility function  $U$ .

The policy functions are depicted in Figure 14. The investment rate  $\iota = I/K$  is decreasing in  $k$ , and is increasing in  $z_t$ . This reflects the adjustment of capital towards its desired value, which is not instantaneous due to the adjustment cost.<sup>25</sup> Note that a positive permanent shock has the effect of lowering  $k$ , since  $K$  is a state variable, which increases investment. Because of permanent shocks, the size distribution is very skewed.

<sup>23</sup>Implicitly, we assume that pure profits (which equal to the entry cost in present value) are counted as capital income in the NIPA.

<sup>24</sup>Except for  $\sigma_m$ , but this does not seem to matter significantly.

<sup>25</sup>The value function is simply an increasing and concave function of  $k$ , and is increasing in  $z_t$ .

### 6.3 Model Experiment (Preliminary)

We consider the following experiment. Assume this model is the truth, and we want to know the effect of a reduction in the adjustment cost parameter from  $\psi = \frac{1}{3}$  to  $\psi = 0$ . An economist fits an AR(1) process with firm fixed effects, using a balanced panel:

$$\log \Pi_{it} = \alpha_i + b \log K_{it} + z_{it}, \quad (15)$$

$$z_{it} = \rho z_{it-1} + \sigma \varepsilon_{it}. \quad (16)$$

For simplicity, assume that the economist knows the correct curvature parameter  $b = \frac{\alpha}{1-v}$ . (Not knowing  $b$  raises additional issues.) Simulations reveal that this economist would estimate  $\rho \simeq .7$  and  $\sigma \simeq .75$ . (These numbers are found by simulating data from the ‘true’ model and estimating ??-16 as the economist would do.)

With these numbers in hand, the economist would predict that reducing adjustment costs from the low level of  $\psi = \frac{1}{3}$  down to 0 would increase average labor productivity (equal to the wage) by 3.48%. However, the true effect is 3.98%, which is markedly bigger. Hence, not knowing the correct process for productivity shocks would lead to a significant error, even if the economist knew exactly the rest of the structure of the economy. These errors are likely to be even larger in some cases, when the persistence matters more, e.g. when adjustment costs are larger or there are additional frictions. (It is easy to construct an example in a model with fixed cost where a firm which faces a temporary shock would never adjust, while a firm facing a permanent shock would adjust.) This cautionary tale thus suggests that the persistence may matter significantly.

## 7 Application: the time variation in idiosyncratic volatility

As an example of application of the methodology, this section studies the variation of idiosyncratic risk over the past 40 years in the United States. This exercise is interesting in the light of two debates. First, some authors argue that idiosyncratic volatility has increased over time (Comin and Philippon, 2005). Figures 6 and 7 reproduce findings of Comin and Philippon that idiosyncratic volatility has trended up in Compustat over this period. (These findings are somewhat sensitive to the sample used.) This is true in the Compustat sample but does not appear to be true in larger Census universes (Davis, Haltiwanger, and Miranda, 2006). Second, several authors have documented that firm-level idiosyncratic risk may be countercyclical (e.g. Bloom 2008, Eisfeldt and Rampini 2007).

The procedure that we used before by pooling cross-section and time-series data can be applied year-by-year, using only cross-sectional moments. Under the assumption that firms realize immediately the new process that they face, we can use purely cross-sectional moments. We can then estimate year-by-year the parameters of the shock process.

Figures 8 and 9 present the evolution of the moments that we use to estimate the model. The evolution of the shock process that we will find is directly due to the evolution of these moments: the variance of profit rates has almost steadily increased, while the variance of investment rates increased in the 1970s before falling back. The serial correlation of investment has increased, while the serial

correlation of profit is mostly trendless. Interestingly, several of these moments show sharp variations in some of the recession years (1981, 2001 in particular).<sup>26</sup> To illustrate the possible cyclicity of these moments, figures 10 and 11 plot hp-filtered log real GDP (annual data) together with the (unfiltered) moments and reports the correlation. The only moment which appears to be significant cyclical is the covariance between profits and average growth over the next three years, which is significantly negative.

These time-varying moments lead to time-varying parameters. I present here only the results of the models A and B (i.e. permanent shock but no AR1 shock and AR1 shock but no permanent shock), which are easier to interpret. Figure 12 gives the results for model A and figure 13 gives the results for model B.

We can see that in both cases, the estimated adjustment cost parameter  $\eta$  increases significantly over time, reflecting a decline in adjustment costs. This seems to be driven, in part, by the decrease in the variance of investment, and in part by the changing correlation between investments and profits. The permanent shock and the iid shock to profits both become large, but they appear to have some important short-run movements as well. The adjustment cost shock increases then falls back, and appears to track the variance of investment.

In the AR1 shock model, the persistence parameter  $\rho$  is trendless. The adjustment cost shock has again a hump-shaped pattern. Both iid and transitory shocks increase over time.

These exercises confirm that in this sample, volatility has increased, and we can trace it down to, essentially, an increase in the variance of permanent shocks (or the highly persistent shock, in the case of the AR1 model). The evidence of reduction of adjustment costs is interesting. Regarding business cycles, there is also a potentially interesting cyclical pattern in the permanent shock: the correlation between GDP and the standard deviation of the permanent shock is  $-0.56$ . This corroborates the view of Bloom (2008), and Eisfeldt and Rampini (2007), that idiosyncratic shocks are countercyclical. Of course, annual data are not ideal to study business cycles.

## 8 Concluding remarks

The shock process is an important, and rather understudied, input into models of firm heterogeneity. The procedure proposed in this paper allows to use investment decisions to infer the persistence of the shock process. The estimates suggest that permanent shocks are important. This calls for more theoretical and quantitative work incorporating permanent shocks into the models, and possibly modeling their sources. For instance, there is little work which integrates the firm size distribution and investment dynamics. Because there is also a substantial component of productivity with a low serial correlation, it also seems important to assess if the results of the literature also hold when productivity is the sum of a permanent and an iid component, rather than an AR(1) component.

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<sup>26</sup>The fact that the correlation between investment and profits is more negative in recessions seems at odds with financial constraints.

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## 9 Data Appendix

To be written.

## 10 Computational Appendix

This appendix sketches the solution method used to solve the general equilibrium model.

(1) Pick the parameters. The process for  $z_t$  is approximated by a Markov chain using Tauchen's method. (I used 6 points.) The (normal) process for  $\varepsilon_p$  is approximated with a finite distribution. (I used 5 points.) We pick a grid for  $k$ . (I used 100 equally spaced points.) Finally we pick a grid for  $\iota$ , the investment rate. (I used 100 points.)

(2) Guess a wage  $w$ .

(3) Iterate until convergence on the Bellman equation, which now has a discrete state and action space:

$$j(k, z_t) = A_2(w)E \left( z_m^{\frac{1}{1-v}} \right) z_t^{\frac{1}{1-v}} k^{\frac{\alpha}{1-v}} + \max_{\iota} \left\{ -(\iota + \phi(\iota, 1))k + \beta(1 - \eta) \sum_{z'_t} Q(z_t, z'_t) \sum_{\varepsilon'_p} \pi(\varepsilon'_p) \left[ e^{\frac{\mu + \sigma_p \varepsilon'_p}{1 - \alpha - v}} j \left( (1 - \delta + \iota) k e^{\frac{-\mu - \sigma_p \varepsilon'_p}{1 - \alpha - v}}, z'_t \right) \right] + \beta \eta k \right\}.$$

(4) Check the free entry condition:

$$\sum_{z_t, z_p} v(z_t, z_p) z_p^{\frac{1}{1-\alpha-v}} j(k_{in}, z_t) = c_{in}.$$



If it does not hold with the required precision, adjust the wage  $w$  and go back to (3).

(5) Compute the policy function  $\iota(k, z_t)$ .

(6) Simulate a large panel of firms, assuming  $N$  firms, with  $\delta N$  randomly picked disappearing each period and replaced by  $\delta N$  new firms. Compute the aggregate labor demand of these firms.

(7) Pick the number of firms in the economy  $M$  to scale the labor demand to  $\bar{N}$ .

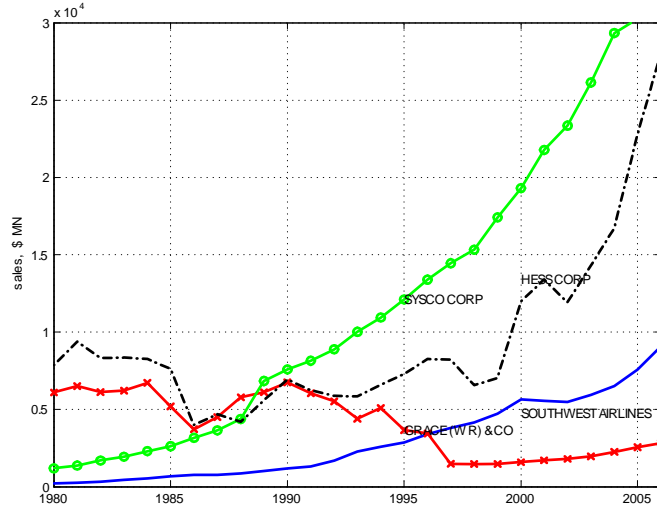


Figure 1: Examples of paths of sales for four firms in the Compustat sample.

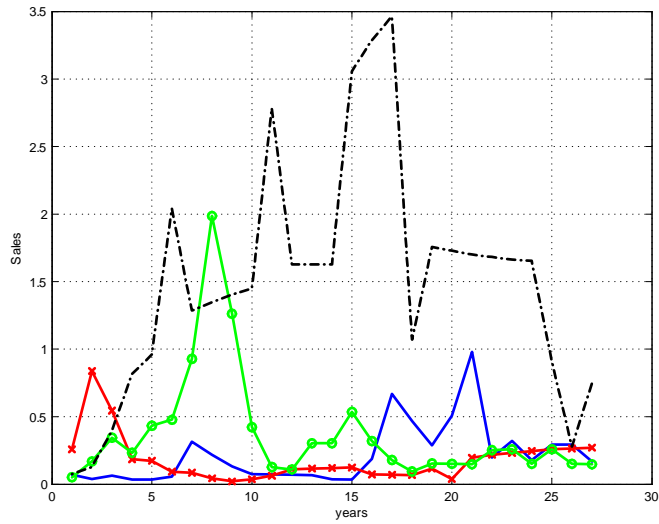


Figure 2: Examples of paths of sales for four firms simulated from a model of industry dynamics.

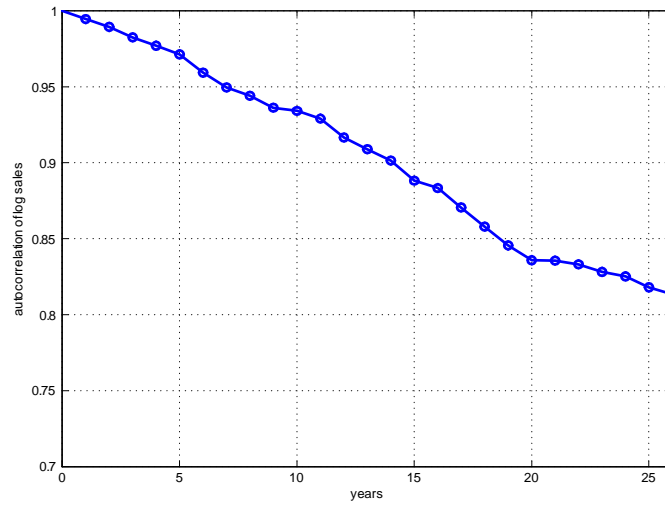


Figure 3: Correlation of log Sales in year  $t$  and in year  $t + k$ , for  $k = 1 \dots 26$ , in the data.

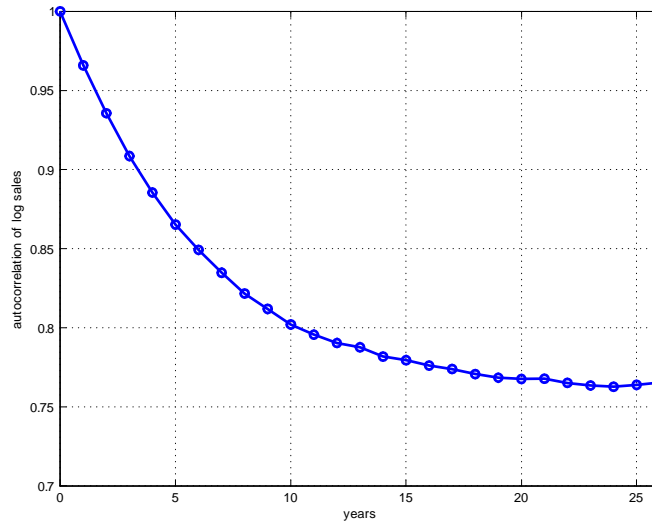


Figure 4: Correlation of log Sales in year  $t$  and in year  $t + k$ , for  $k = 1 \dots 26$ , in an industry model similar to Gomes (2001).

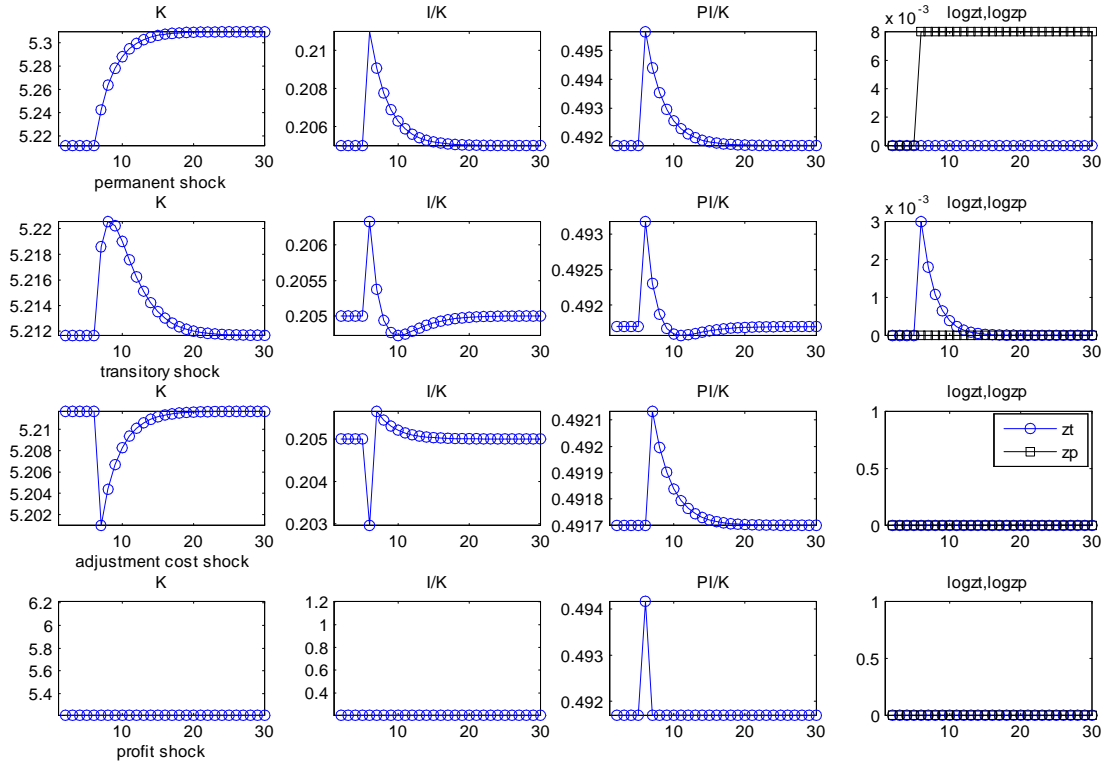


Figure 5: Impulse responses to the four shocks. The first row depicts the response to a permanent shock to productivity  $\varepsilon^P$  at time  $t = 5$ . The second, third and fourth rows depict the responses to a transitory shock  $\varepsilon^T$ , an adjustment cost shock  $\varepsilon^{AC}$ , and an iid profit shock  $\varepsilon^M$ .

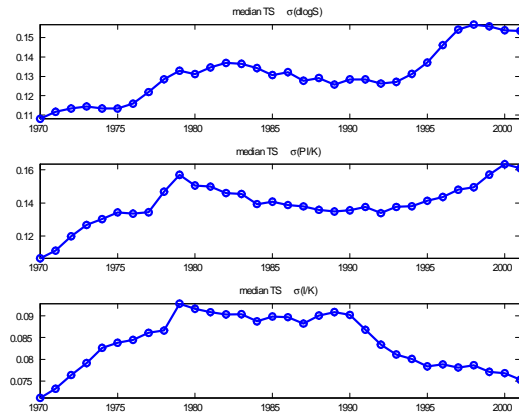


Figure 6: This figure plots the median across firm of the time-series volatility of sales, investment rate and profit rate. The time-series volatility is computed at each date through a 11 year centered window. Sample: unbalanced.

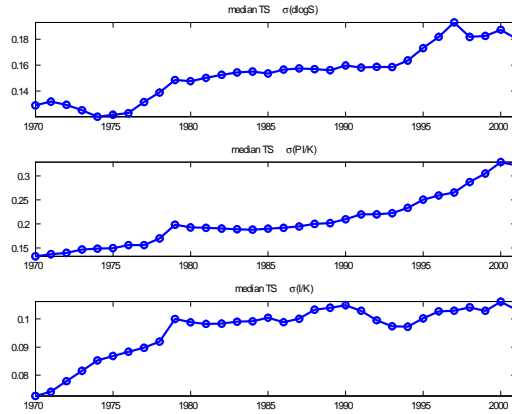


Figure 7: This figure plots the median across firm of the time-series volatility of sales, investment rate and profit rate. The time-series volatility is computed at each date through a 11 year centered window. Sample: all of Compustat except missing data on sales (resp. investment rate, profit rate).

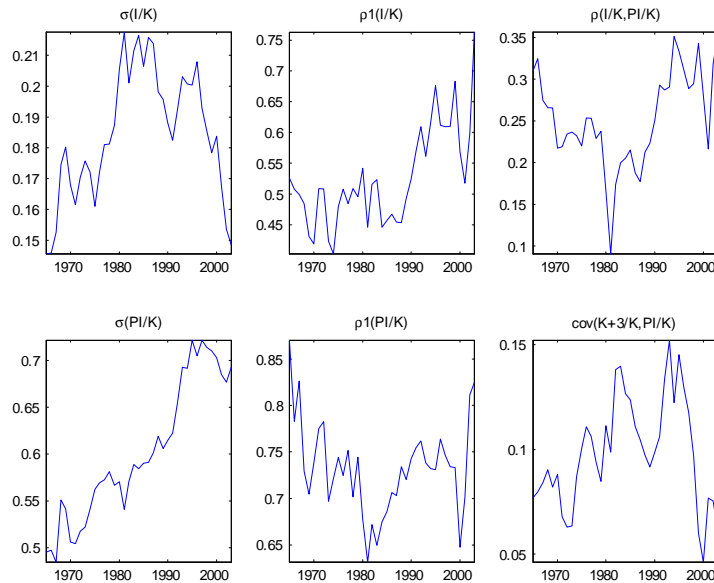


Figure 8: This figure plots, for each date (1965 to 2003) the estimated standard deviation of investment rate, of profit rate, the autocorrelation of the profit rate and of the investment rate, as well as the correlation between the investment rate and the profit rate, and the covariance between the current profit rate and the next 3y growth rate of capital.

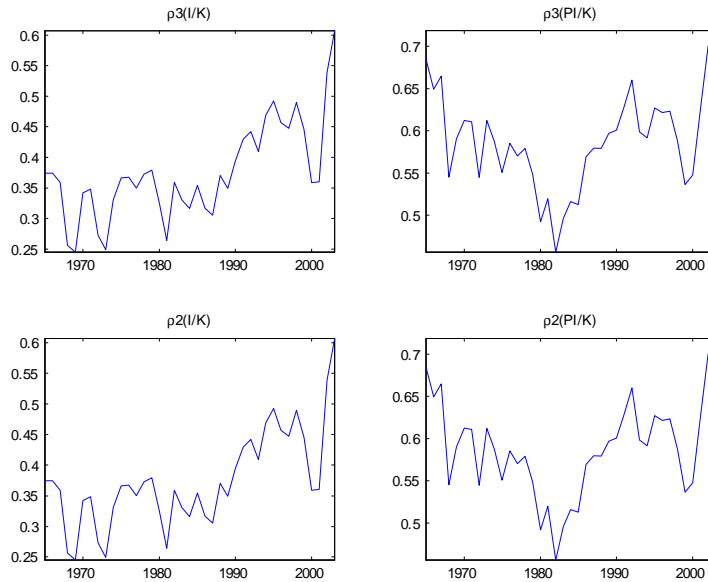


Figure 9: This figure plots, for each date (1965 to 2003) the estimated autocorrelation of the profit rate (2 lags and 3 lags) and autocorrelation of the investment rate (2 lags and 3 lags).

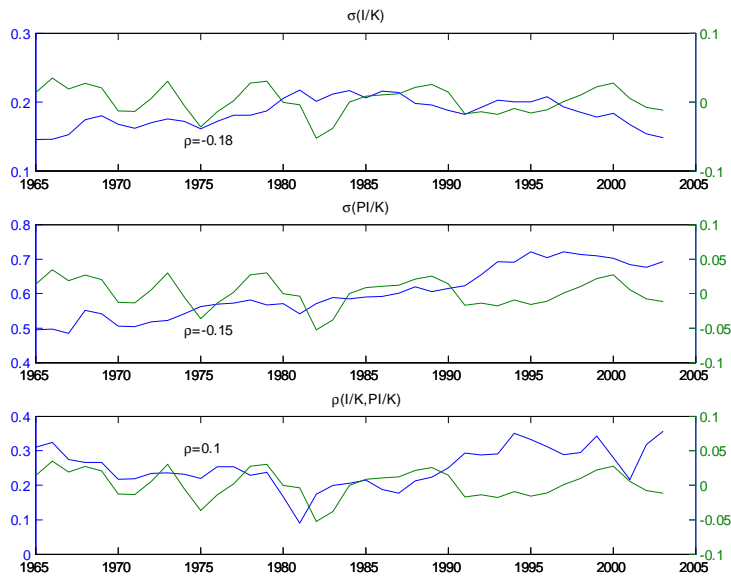


Figure 10: This figure plots, for each date (1965 to 2003) the estimated standard deviation of the investment rate, of the profit rate, and the correlation between the investment rate and the profit rate. The green line is real GDP (hp filtered). The correlation between each series and GDP is displayed on each graph.

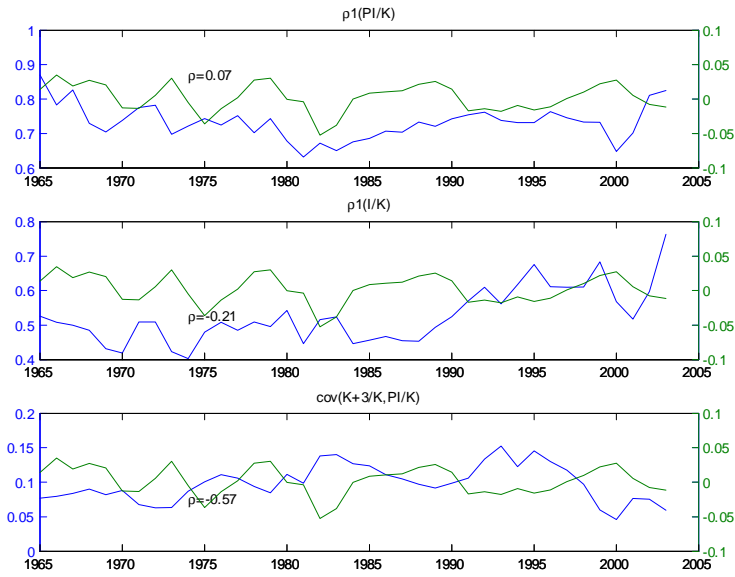


Figure 11: This figure plots, for each date (1965 to 2003) the estimated autocorrelation of the profit rate, autocorrelation of the investment rate, and the covariance between the profit rate and 3y future capital growth. The green line is real GDP (hp filtered). The correlation between each series and GDP is displayed on each graph.

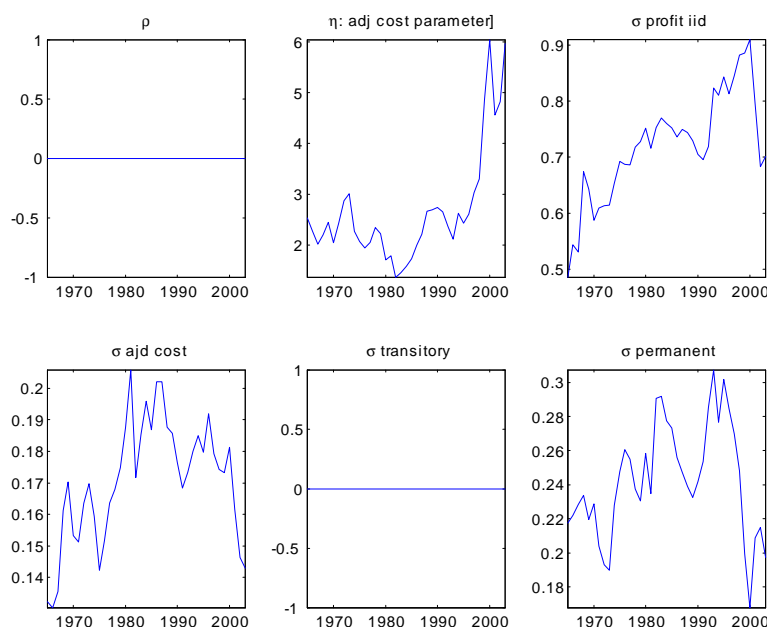


Figure 12: Estimated parameters: case of time-varying parameters (section 7). In each year, I repeat the estimation exercise using the cross-sectional moments of that year. Estimates from the model with permanent shocks but no AR1 shocks.

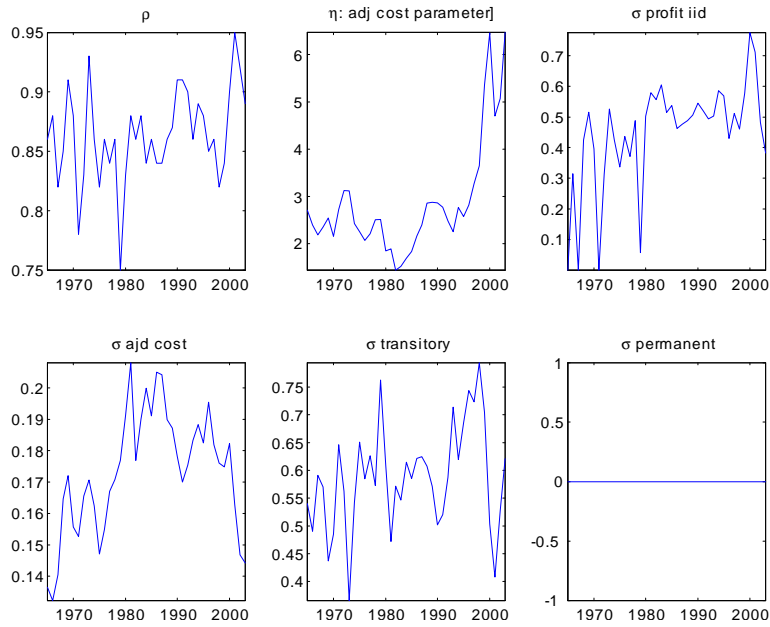


Figure 13: Estimated parameters: case of time-varying parameters (section 7). In each year, I repeat the estimation exercise using the cross-sectional moments of that year. Estimates from the model with AR1 shocks but no permanent shocks.

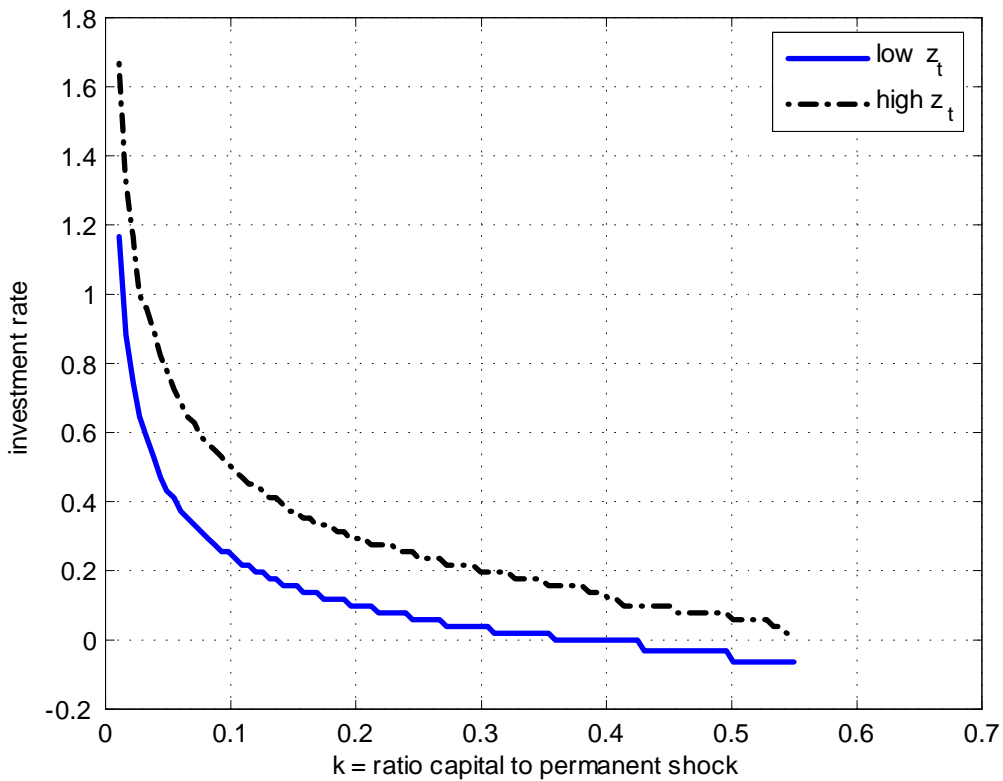


Figure 14: This figure plots the policy function in the GE model of Section 6: this is the investment rate  $I/K$ , which is a function of  $k$  (capital scaled by the permanent shock) and  $z_t$ , the temporary productivity.