An Empirical Model of Wage Dispersion with Sorting

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PRELIMINARY

Abstract

The paper studies contributions to wage dispersion in a model that allows for sorting in firm-worker matches. The model is a general equilibrium on-the-job search model with wage formation similar to that of Cahuc, Postel-Vinay, and Robin (2006). Workers differ in their permanent skill level and firms differ with respect to productivity. As shown in Lentz (2007), in this setting, positive (negative) sorting results if the match production function is supermodular (submodular). If the production function is modular, no sorting obtains. The model is estimated by indirect inference on Danish matched employer-employee data using extensive information about both workers and firms. In addition to observations on wage variation and worker flows, the estimation also emphasizes firm level output data. The auxiliary model includes the wage variance decomposition of Abowd, Kramarz, and Margolis (1999) (AKM). We find that while the AKM estimates of worker and firm types are biased for our model, it is nevertheless a useful source of identifying variation and in many cases correctly captures the sign of the correlation between worker and firm types.

1 Introduction

Why are seemingly identical workers paid differently? Mincerian wage equation regressions that attempt to explain wage variation with observed variation in worker characteristics typically only explain 25-30% of the observed variation in wages across workers. In this paper, wage dispersion refers to the variation in the residual of such Mincerian wage equations.

Sorting may play an important role as a source of wage dispersion. Clearly, a given distribution of worker and firm types can produce very different output and wage distribution outcomes depending on how matches are formed. Previous work on the estimation of sources of wage dispersion in Abowd, Kramarz, and Margolis (1999), Postel-Vinay and Robin (2002), and Cahuc, Postel-Vinay, and Robin (2006) has adopted the maintained and identifying assumption that match formation is independent of the types of the agents involved. The analysis of Abowd, Kramarz, and Margolis (1999) directly estimates individual worker and firm fixed effects. Subsequent to the estimation the authors test whether the estimated fixed effects are correlated in the data and find little correlation. This has been taken as evidence that sorting is not an important issue in the labor market. It is however problematic to test the hypothesis of sorting within a framework where the maintained identifying assumptions rule out key mechanisms that can produce sorting in models with production function complementarities.

This paper puts forth a general equilibrium on-the-job search model with both firm and worker heterogeneity. The analysis is based on an off-the-shelf model of on-the-job search with endogenous search intensity as in Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005) combined with wage determination as in Cahuc, Postel-Vinay, and Robin (2006). The model is analyzed in detail in Lentz (2007). Depending on the production function, the worker's search intensity can be type dependent and sorting will result. The subsequent empirical analysis will decompose wage dispersion into 4 sources; worker heterogeneity, firm heterogeneity, friction, and sorting. Postel-Vinay and Robin (2002) decompose dispersion into the first three components.

Abowd, Kramarz, and Margolis (1999) perform a decomposition of observed wage dispersion in French matched employer-employee data into unobserved worker and firm fixed effects. The panel structure of such data sets allows the continued observation of a single worker matched with

different employers which is the basis of identification of individual fixed effects. The identification of the fixed effects is done under the maintained assumption that job transitions are not correlated with either worker or firm fixed effects. This precludes natural sorting mechanisms in job transition models.

Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay, and Robin (2006) make the point that the identification of unobserved fixed effects in Abowd, Kramarz, and Margolis (1999) can be biased in the presence of frictions. Specifically, the contribution of worker fixed effect dispersion to overall wage dispersion can be upward biased if the estimation does not specifically control for the particular properties of the wage process in an on-the-job search model. In these papers, wage dispersion is explained through a structural estimation of a general equilibrium on-the-job search model. Friction is given a role separate from dispersion in worker and firm effects in the explanation of overall wage dispersion. Both of the papers assume that the distribution of worker types is independent of the type of the firm. The production function in these papers is supermodular. However, the matching technology is assumed such that sorting does not arise. This paper proposes a search technology where sorting can arise in response to production function complementarities. The theoretical aspects of the model are described in greater detail in Lentz (2007).

2 Model

The framework of the model is an endogenous search intensity model with type heterogeneity on both the worker and firm side. The paper adopts the wage determination mechanisms of Cahuc, Postel-Vinay, and Robin (2006).

There is a continuum of firms and potential entrants with measure m, and a continuum of workers with measure normalized at unity. A worker is characterized by his or her permanent innate ability h which is independently and identically distributed across workers according to the cumulative distribution function $\Psi(\cdot)$. Firms differ with respect to their permanent productivity realization p which is independently and identically distributed across firms according to the cumulative distribution function $\Phi(\cdot)$.

Workers can be either employed or unemployed. Regardless of employment state, a worker

can search for a new job. The analysis will allow that the search technology may differ across the two employment states. Specifically, a search intensity s results in the arrival rate of new job opportunities of $(\mu + \kappa s)\lambda(\theta)$ or $s\lambda(\theta)$ if unemployed or employed, respectively, where $\kappa > 0$. If $\kappa > 1$ then search is more efficient in the unemployed state. $\mu \geq 0$ represents an arrival of offers that is unrelated to the search decision of the worker. $\lambda(\theta)$ is the equilibrium arrival rate of offers per search unit and θ is market tightness. By assumption $\lambda'(\theta) \geq 0$. The cost of a search intensity s is given by the increasing and convex function,

$$c(s) = \frac{c_0 s^{1+\frac{1}{c_1}}}{1+\frac{1}{c_1}}, \tag{1}$$

where $c_0 > 0$ is a scale parameter and $c_1 > 0$ sets curvature.

A match between a type h worker and a type p firm produces value added $f(p,h)$ net of payments to capital inputs. It is assumed that $f_p(h,p) \geq 0$ and $f_h(h,p) \geq 0$ for all (h,p). Hence, more skilled workers enjoy an absolute advantage relative to less skilled workers regardless of the firm type p they are matched with. Likewise for the ranking of firms. Hence, the labels by which types are indexed, h and p, define unambiguous rankings such that a high h indicates a placement in the top of the worker skill ranking and a high p value indicates a top placement in the firm productivity ranking. Statements on sorting then become statements about match allocation patterns in terms of worker skill and firm productivity rankings. We adopt the particular production function specification,

$$f(h,p) = f_0\big(\alpha h^\rho + (1-\alpha)p^\rho\big)^{\frac{1}{\rho}}, \tag{2}$$

where f_0 is a scale parameter, and $0 \leq \alpha \leq 1$. If $\rho < 1$, then the production function is supermodular. It is submodular for $\rho > 1$. The production function is modular for $\rho = 1$. As shown in Lentz (2007) if the production function is supermodular, the equilibrium will be characterized by positive sorting between worker skill and firm productivity. If it is submodular, negative sorting will result. For $\rho = 1$ there will be no sorting between worker skill and firm productivity types.

For the sake of simplicity, the capital share is assumed constant across matches. Hence, the capital cost for a given match is $K(p,h) = kf(h,p)$, where $0 \leq k < 1$. Therefore, the capital share is $k/(1+k)$. In the following analysis of match formation, values are stated in terms of the surplus net of capital costs.

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Match separation occurs as the result of one of three distinct events. First, the worker in the match may receive an offer from an outside firm with greater productivity than the current firm which induces a quit. Second, at rate $\delta_0 \lambda(\theta)$ the worker makes a job-to-job transition where the new job is drawn randomly from the vacancy offer distribution and the outside option in the new job is unemployment. The process is meant to capture that a substantial number of job-to-job transitions are observably not up the offer ladder.[1] One possible explanation is that a, to the econometrician, unobserved shock has reduced the worker's valuation of the current match which induces a job-to-job transition. Nagypál (2005) provides an explicit argument for such a process. It may also be that the worker has been given notice of a lay-off sufficiently far in advance that the worker was able to obtain a new job without an actual unemployment spell in between. The model does not take an explicit stand on the exact source of the shock. It simply allows that exogenous match separations can occur where the worker's climb up the offer ladder is reset but without the association of an actual unemployment spell. Third, at exogenous rate δ_1 the match is destroyed and the worker moves into unemployment.

Employment contracts between workers and employers are set through a Rubinstein (1982) style bargaining game following the same protocol as in Cahuc, Postel-Vinay, and Robin (2006). An alternative bargaining protocol is presented in Yamaguchi (2006). In both cases, it is assumed that the worker can use a contact with one employer as a threat point in a bargaining game with another. An employment contract can only be re-negotiated by mutual consent. If the worker is unemployed, then the value of unemployment will be the worker's threat point. The detailed bargaining argument is presented in the appendix.

An employment contract consists of a worker's wage level and search intensity. Specifically, this implies the assumption that search intensities can be contracted upon. In the current setting this assumption implements the jointly efficient search intensity level. In the alternative case where search intensities are chosen by the worker in response to some match surplus split, the worker's search intensity will be inefficiently high in the case where the worker is not receiving the full surplus of the match. As such, one can think of the setup in this paper as describing an upper

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[1]Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005) and Nagypál (2005) emphasize that this type of separation shock is empirically important.

bound on the efficiency of search in the model.

Denote by $\tilde{V}(h, p, w, s)$ a type h worker's asset value of a job with a type p firm and employment contract (w, s). The outcome of the employment contract bargaining as described in the appendix is such that the agreed upon search intensity maximizes the joint surplus of the match and the wage then dictates the surplus split. Hence, the search intensity depends only on the (h, p) pair,

$$s(h, p) = \arg \max_{s \geq 0} \tilde{V}\big(h, p, f(h, p), s\big). \tag{3}$$

If the worker is unemployed, the outside option in the bargaining is the value of unemployment. Denote by $\big(w_0(h, p), s(h, p)\big)$ the employment contract of a type h worker who was hired out of unemployment by a type p firm. It satisfies,

$$\tilde{V}(h, w_0(h, p), p, s(h, p)) = \beta \tilde{V}\big(h, f(h, p), p, s(h, p)\big) + (1 - \beta)V_0(h), \tag{4}$$

where $V_0(h)$ is the asset value of unemployment for a type h worker. β is the worker's bargaining power.

If an employed worker receives an outside offer, the worker will go to the most productive firm and the outcome is as if the worker bargains with the most productive firm with a threat point of going to the less productivity firm and receive full surplus. Denote by p and q the types of the two firms, where $p \geq q$. If the two firms are of equal productivity, the worker stays with the current firm. Denote the resulting wage by $w(h, q, p)$. It satisfies,

$$\tilde{V}(h, p, w(h, q, p), s(h, p)) = \beta \tilde{V}\big(h, p, f(h, p), s(h, p)\big) + (1 - \beta)\tilde{V}\big(h, q, f(h, q), s(h, q)\big). \tag{5}$$

Denote by $q(h, p, w)$ the highest type a worker who is currently employed by a type p firm at wage w such that the meeting has no impact on the current employment terms. It is defined implicitly by,

$$w = w\big(h, p, q(h, p, w)\big). \tag{6}$$

This implies that,

$$\tilde{V}\big(h, p, w, s(h, p)\big) = \beta \tilde{V}\big(h, p, f(h, p), s(h, p)\big) + (1 - \beta)\tilde{V}\big(h, q, f(h, q), s(h, q)\big), \tag{7}$$

where $q = q(h, p, w)$. Equation (6) also illustrates that any arbitrary wage received in a match with a type p firm can be viewed as the outcome of bargaining with the type p firm given the outside

option to match with a type $q(h, p, w)$ firm. Hence, a sufficient statistic for an employed worker's state is the record of the types of the two most productive employers that the worker has met during the past employment spell. Mostly, the value functions in the following will be stated in these terms rather than through an explicit wage. Specifically define $V(h, q, p) = \tilde{V}(h, p, w(h, q, p), s(h, p))$.

It is assumed that an unemployed type h worker receives an income stream $f(h, b)$. The Bellman equation for the value of unemployment is given by,

$$
\begin{aligned}
rV_0(h) &= \max_{s \geq 0} \left\{ f(h, b) - c(s) + (\mu + \kappa s)\lambda(\theta) E \left[\max\left[0, \tilde{V}(h, w_0(h, p), p) - V_0(h) \right] \right] \right\} \\
&= \max_{s \geq 0} \left\{ f(h, b) - c(s) + (\mu + \kappa s)\lambda(\theta) \int_{R(h)}^{\bar{p}} \beta \left[V(h, p', p') - V_0(h) \right] d\Gamma(p') \right\},
\end{aligned}
\tag{8}
$$

where r is the interest rate, $\Gamma(p)$ is the cumulative firm type vacancy distribution, and $R(h)$ is the type h reservation productivity level defined by,

$$
V\big(h, R(h), R(h)\big) = V_0\big(h\big).
\tag{9}
$$

It is straightforward to prove that $V(h, p, p)$ is monotonically increasing in p which establishes the reservation property of the model; that a type h worker will agree to match with any employer above the productivity threshold level, $R(h)$. Applying integration by parts and the envelope theorem, equation (8) can be restated as,

$$
rV_0(h) = \max_{s \geq 0} \left\{ f(h, b) - c(s) + (\mu + \kappa s)\lambda(\theta) \int_{R(h)}^{\bar{p}} \frac{\beta f'_p(h, p')[1 - \Gamma(p')]dp'}{r + \delta + \beta s(h, p')\lambda(\theta)[1 - \Gamma(p')]} \right\},
\tag{10}
$$

where $\delta \equiv \delta_0 \lambda(\theta) + \delta_1$.

The value of employment with a type p firm at wage $w(h, q, p)$ and search intensity $s(h, p)$ is given by,

$$
\begin{aligned}
rV(h, q, p) &= w(h, q, p) - c\big(s(h, p)\big) + \delta_1 \big[V_0(h) - V(h, q, p)\big] + \\
&\quad s(h, p)\lambda(\theta) \left[\int_p^{\bar{p}} \big[V(h, p, p') - V(h, q, p)\big] d\Gamma(p') + \int_q^p \big[V(h, p', p) - V(h, q, p)\big] d\Gamma(p') \right] + \\
&\quad \delta_0 \lambda(\theta) \left[\Gamma(R(h))V_0(h) + \int_{R(h)}^{\bar{p}} V(h, R(h), p') d\Gamma(p') - V(h, q, p) \right].
\end{aligned}
\tag{11}
$$

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Integration by parts and the envelope theorem allows the expression to be re-written as,

$$
\begin{aligned}
(r+\delta)V(h,q,p) \ = \ & w(h,q,p) - c(s(h,p)) + \delta V_0(h) + \delta_0\lambda(\theta)\int_{R(h)}^{\bar{p}}\frac{\beta f_p(h,p')\big(1-\Gamma(p')\big)dp'}{r+\delta+\beta s(h,p')\lambda(\theta)\big(1-\Gamma(p')\big)} + \\
& s(h,p)\lambda(\theta)\int_{p}^{\bar{p}}\frac{\beta f_p(h,p')\big(1-\Gamma(p')\big)dp'}{r+\delta+\beta s(h,p')\lambda(\theta)\big(1-\Gamma(p')\big)} + \\
& s(h,p)\lambda(\theta)\int_{q}^{p}\frac{(1-\beta)f_p(h,p')\big(1-\Gamma(p')\big)dp'}{r+\delta+\beta s(h,p')\lambda(\theta)\big(1-\Gamma(p')\big)}.
\end{aligned}
\tag{12}
$$

The detailed derivation of equation (12) can be found in the appendix.

2.1 The search choices

The employment state conditional search intensity is found by use of equations (3) and (8). Together with equation (12), they imply the first order conditions,

$$
c'(s_0(h)) \ = \ \kappa\lambda(\theta)\int_{R(h)}^{\bar{p}}\frac{\beta f_p(h,p')\big(1-\Gamma(p')\big)dp'}{r+\delta+\beta s(h,p')\lambda(\theta)\big(1-\Gamma(p')\big)}
\tag{13}
$$

$$
c'\big(s(h,p)\big) \ = \ \lambda(\theta)\int_{p}^{\bar{p}}\frac{\beta f_p(h,p')\big(1-\Gamma(p')\big)dp'}{r+\delta+\beta s(h,p')\big(1-\Gamma(p')\big)}.
\tag{14}
$$

By convexity of $c(\cdot)$, differentiation of equation (14) with respect to p immediately yields that $s(h,p)$ is monotonically decreasing in p, $\forall h$. Furthermore, $s(h,\bar{p})=0$, $\forall h$. Lemma 1 establishes that the search intensity is strictly increasing in the worker type h if the production function is strictly supermodular. Also, if the production function has no complementarities between worker and firm types, then the search intensity is identical across worker types.

Lemma 1 *For any pair $(h_0,h_1)\in[\underline{h},\bar{h}]\times[\underline{h},\bar{h}]$ such that $h_0<h_1$, and for all $p\in[b,\bar{p})$,*

- *$f_{hp}(h,p)>0\,\forall(h,p)\Rightarrow s(h_0,p)<s(h_1,p)$ (supermodular).*

- *$f_{hp}(h,p)<0\,\forall(h,p)\Rightarrow s(h_0,p)>s(h_1,p)$ (submodular).*

- *$f_{hp}(h,p)=0\,\forall(h,p)\Rightarrow s(h_0,p)=s(h_1,p)$ (modular).*

For any $h\in[\underline{h},\bar{h}]$, $s(h,\bar{p})=0$.

Proof. See Lentz (2007) ∎

The reservation productivity level $R(h)$ defined in equation (9) is characterized in Lemma 2

Lemma 2 *For any $h \in [\underline{h}, \bar{h}]$, if $\kappa = 1$ and $\mu = \delta_0$ then $R(h) = b$, and if $\kappa > 1$ and $\mu > \delta_0$ then $\bar{p} > R(h) > b$. Furthermore, if for any pair $(h_0, h_1) \in [\underline{h}, \bar{h}]$ and for all $p \in [b, \bar{p}]$ $f_p(h_0, p) = f_p(h_1, p)$, then $R(h_0) = R(h_1)$.*

Proof. See Lentz (2007) ∎

In the case where $\kappa > 1$, an obvious question of interest is how $R(h)$ varies with h. Lemma 2 states that in the absence of production function complementarities, $R(h)$ is identical across worker types. If $\rho \neq 1$ the model includes many of the complications associated with the classic stopping problem as analyzed in Shimer and Smith (2000). Specifically, it is straightforward to produce examples where $R(h)$ is not monotonically increasing in h even if the production function is supermodular.

2.2 Solving for the wage

With a solution for $s(h, p)$ in hand, one can immediately obtain values for the Bellman equation for the following states,

$$(r + \delta)V(h, p, p) = f(h, p) - c(s(h, p)) + \delta V_0(h) + \delta_0 \lambda(\theta) \int_{R(h)}^{\bar{p}} \frac{\beta f_p(h, p')(1 - \Gamma(p'))dp'}{r + \delta + \beta s(h, p')\lambda(\theta)(1 - \Gamma(p'))} +$$
$$s(h, p)\lambda(\theta) \int_p^{\bar{p}} \frac{\beta f_p(h, p')(1 - \Gamma(p'))dp'}{r + \delta + \beta s(h, p')\lambda(\theta)(1 - \Gamma(p'))}, \; \forall p \geq b. \tag{15}$$

The value of the unemployed state is,

$$rV_0(h) = f(h, b) - c(s_0(h)) + (\mu + \kappa s_0(h))\lambda(\theta) \int_{R(h)}^{\bar{p}} \frac{\beta f_p(h, p')(1 - \Gamma(p'))dp'}{r + \delta + \beta s(h, p')\lambda(\theta)(1 - \Gamma(p'))}. \tag{16}$$

Given the wage determination mechanism in equation (5) combined with equation (15), one obtains,

$$V(h, q, p) = \beta V(h, p, p) + (1 - \beta)V(h, q, q). \tag{17}$$

It then directly follows from equation (12) that,

$$w(h, q, p) = (r + \delta)V(h, q, p) + c(s(h, p)) - \delta V_0(h) - \delta_0 \lambda(\theta) \int_{R(h)}^{\bar{p}} \frac{\beta f_p(h, p')(1 - \Gamma(p'))dp'}{r + \delta + \beta s(h, p')\lambda(\theta)(1 - \Gamma(p'))} -$$
$$s(h, p)\lambda(\theta) \left[\int_p^{\bar{p}} \frac{\beta f_p(h, p')(1 - \Gamma(p'))dp'}{r + \delta + \beta s(h, p')\lambda(\theta)(1 - \Gamma(p'))} + \int_q^p \frac{(1 - \beta)f_p(h, p')(1 - \Gamma(p'))dp'}{r + \delta + \beta s(h, p')\lambda(\theta)(1 - \Gamma(p'))} \right]. \tag{18}$$

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2.3 Vacancy creation

Each firm is characterized by a permanent productivity p that applies to all of its matches. Firm types are distributed according to the cumulative distribution function $\Phi(\cdot)$. A firm's total output is the sum of the output of all its matches. Hence, a firm with n workers produces,

$$Y(h^n, p) = \sum_{i=1}^{n} f(h_i, p).$$

The total wage bill of the firm depends not only on the vector of worker types, but also on the next best offer of each worker.

At any given time, each firm chooses a vacancy intensity ν at cost $c_\nu(\nu)$, where $c_\nu(\cdot)$ is strictly increasing and convex. Given the choice of vacancy intensity, the firm meets a new worker at rate $\eta\nu$. If a productivity p firm meets a skill h worker currently matched with a productivity $p' < p$ firm, the worker will accept to match with the productivity p firm. The bargaining will award value $V(h, p', p)$ to the worker and the firm will receive value $V(h, p, p) - V(h, p', p)$, which is the full match surplus minus the worker's share. The vacancy intensity choice is made so as to maximize the value of the firm's hiring operation,

$$J_0(p) = \max_{\nu \geq 0} \left[-c_\nu(\nu) + \eta\nu \int_{\underline{h}}^{\bar{h}} \int_{R(h')}^{p} \left[V(h', p, p) - V(h', p', p) \right] d\Lambda(h', p') \right], \tag{19}$$

where

$$\Lambda(h, p) = \frac{\int_{\underline{h}}^{h} \left\{ \frac{u}{1-u} [\mu + \kappa s_0(h')] \upsilon(h') + \delta_0 \int_{b}^{\bar{p}} g(h', p') dp' + \int_{b}^{p} s(h', p') g(h', p') dp' \right\} dh'}{\int_{\underline{h}}^{\bar{h}} \left\{ \frac{u}{1-u} [\mu + \kappa s_0(h')] \upsilon(h') + \int_{b}^{\bar{p}} [\delta_0 + s(h', p')] g(h', p') dp' \right\} dh'}$$

is the cumulative distribution function of searching workers by skill level and productivity of the firm in the current match. For the purpose of the definition of Λ, unemployment is considered as a match at the reservation level. The expression reflects a proportionality assumption in matching; a worker is represented in the pool of searchers proportionally to his or her search intensity. $g(h, p) = \int_{b}^{p} g(h, q, p) dq$ is the density of matches between skill h workers and productivity p firms, where $g(h, q, p)$ is the joint pdf of matches. u is the unemployment rate and $\Upsilon(h)$ is the CDF of worker skill in the unemployment pool.

It follows from equation (19) that the first order condition on the productivity conditional

vacancy intensity choice is,

$$c'_\nu\left(\nu\left(p\right)\right) = \eta\left(1-\beta\right)\int_{\underline{h}}^{\bar{h}}\int_{R(h')}^{p}\left[V\left(h',p,p\right) - V\left(h',p',p'\right)\right]d\Lambda\left(h',p'\right). \tag{20}$$

A firm's hiring rate is the product of the meeting rate and the probability that the worker in question accepts the firm's offer,

$$\eta(p) = \eta\nu(p)\int_{\underline{h}}^{\bar{h}} I(R(h') \le p)d\Lambda(h',p). \tag{21}$$

The expected match separation rate for a type p firm is given by,

$$d\left(p\right) = \delta + \left[1-\Gamma\left(p\right)\right]\frac{\int_{\underline{h}}^{\bar{h}} s\left(h,p\right)g\left(h,p\right)dh}{\int_{\underline{h}}^{\bar{h}} g\left(h,p\right)dh}. \tag{22}$$

2.4 Steady state

The steady state condition on the joint CDF of matches, $G(h,q,p)$, is,

$$(1-u)\delta G(h,q,p) + (1-u)\int_{\underline{h}}^{h}\int_{R(h')}^{q}\lambda(\theta)\Bigg\{\left(1-\Gamma(p)\right)\int_{q'}^{q} s\left(h',p'\right)dG\left(h',q',p'\right)$$

$$+\left(1-\Gamma(q)\right)\int_{q}^{p} s\left(h',p'\right)dG\left(h',q',p'\right)\Bigg\} \quad =$$

$$\int_{\underline{h}}^{h} I(R(h') \le q)\lambda(\theta)\left[\Gamma(p) - \Gamma(R(h'))\right]\Bigg[u[\mu + \kappa s_0(h')]\upsilon(h') +$$

$$(1-u)\delta_0\int_{R(h')}^{\bar{p}}\int_{q'}^{\bar{p}} g(h',q',p')dp'dq'\Bigg]dh', \tag{23}$$

where $I(\cdot)$ is an indicator function that equals one if its expression is true, zero if false. The left hand side captures the flow out of the $G(h,q,p)$ mass and the right hand side is the flow in. By steady state the two flows must equal each other. Equation (23) implies that steady state unemployment satisfies,

$$u = \left[\int_{\underline{h}}^{\bar{h}}\left(1 + \frac{[1-\Gamma(R(h'))][\mu + \kappa s_0(h')]\lambda(\theta)}{\delta_0\lambda(\theta)\Gamma(R(h')) + \delta_1}\right)d\Upsilon(h')\right]^{-1}. \tag{24}$$

Using equation (24), one can re-write equation (23) as (see the detailed derivations in the appendix),

$$\int_{\underline{h}}^{h}\int_{R(h')}^{q}\Bigg[\int_{q'}^{q}[\delta/\lambda(\theta) + [1-\Gamma(p)]s(h',p')]g\left(h',q',p'\right)dp'$$

$$+\int_{q}^{p}[\delta/\lambda(\theta) + [1-\Gamma(q)]s(h',p')]g(h',q',p')dp'\Bigg]dq'dh' \quad =$$

$$\frac{\delta}{\lambda(\theta)}\frac{\int_{\underline{h}}^{h} I(R(h') \le q)[\Gamma(p) - \Gamma(R(h'))]\frac{\mu + \kappa s_0(h')}{\delta_0\Gamma(R(h')) + \delta_1/\lambda(\theta)}d\Psi(h')}{\int_{\underline{h}}^{\bar{h}}\frac{[1-\Gamma(R(h'))][\mu + \kappa s_0(h')]}{\delta_0\Gamma(R(h')) + \delta_1/\lambda(\theta)}d\Psi(h')}. \tag{25}$$

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In steady state, the mass of productivity p firms with n workers $m_n(p)$ must be constant. Hence, the steady state firm size distribution satisfies,

$$0 = \eta(p) m_{n-1}(p) + d(p)(n+1)m_{n+1}(p) - (\eta(p) + d(p)n) m_n(p), \tag{26}$$

for all $n \geq 1$ and p. It can be shown that the firm's expected labor force composition is independent of its size. Hence, the expected destruction rate of matches is $d(p)$ for any firm size. Also, in steady state the number of firm births must equal the number of deaths,

$$\eta(p)m_0(p) = d(p)m_1(p). \tag{27}$$

Furthermore, it is given that

$$\sum_{n=0}^{\infty} m_n(p) = m\phi(p), \tag{28}$$

where $\phi(p)$ is the firm productivity distribution pdf. Equations (26)-(28) imply that the type conditional firm size distribution $m_n(p)/(m\phi(p))$ is Poisson with arrival rate $\eta(p)/d(p)$,

$$m_n(p) = \left(\frac{\eta(p)}{d(p)}\right)^n \frac{1}{n!} \exp\left(-\frac{\eta(p)}{d(p)}\right) m\phi(p), \tag{29}$$

for all $n \geq 0$.

2.5 Steady state equilibrium

The equilibrium vacancy offer distribution is given by,

$$\Gamma(p) = \frac{\int_b^p \nu(p') \, d\Phi(p')}{\int_b^{\bar{p}} \nu(p') \, d\Phi(p')}. \tag{30}$$

In equilibrium, the meeting rates of both workers and firms must balance which implies,

$$\lambda(\theta) = \theta\eta(\theta), \tag{31}$$

where

$$\theta = \frac{m \int_b^{\bar{p}} \nu(p') \, d\Phi(p')}{u \int_{\underline{h}}^{\bar{h}} [\mu + \kappa s_0(h)] d\Upsilon(h) + (1-u) \int_{\underline{h}}^{\bar{h}} \int_b^{\bar{p}} [\delta_0 + s(h,p)] dG(h,p)}. \tag{32}$$

Furthermore, the overall worker type distribution is related to the employment state conditional type distributions by, $\Psi(h) = (1-u)G(h,\bar{p},\bar{p}) + u\Upsilon(h)$ which by use of the steady state conditions

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on $G(h,q,p)$ and u produces (see detailed derivations in the appendix),

$$\Upsilon(h) = \frac{\int_{\underline{h}}^{h} \frac{\delta_0 \Gamma(R(h')) + \delta_1/\lambda(\theta)}{\delta_0 \Gamma(R(h')) + \delta_1/\lambda(\theta) + [1 - \Gamma(R(h'))][\mu + \kappa s_0(h')]} d\Psi(h')}{\int_{\underline{h}}^{\bar{h}} \frac{\delta_0 \Gamma(R(h')) + \delta_1/\lambda(\theta)}{\delta_0 \Gamma(R(h')) + \delta_1/\lambda(\theta) + [1 - \Gamma(R(h'))][\mu + \kappa s_0(h')]} d\Psi(h')}. \tag{33}$$

With these conditions, steady state equilibrium can be defined.

Definition 1 *A steady state equilibrium is a tuple* $\{G(h,q,p), \Upsilon(h), \Gamma(p), u, s(h,p), s_0(h), R(h), \eta,$
$w(h,q,p)\}$ *that satisfies equations (9), (13), (14), (18), (24), (25), (30), (32), and (33).*

Lentz (2007) provides proof of existence and uniqueness of equilibrium in a slightly simpler version of the model where vacancy intensities are constant across firm types.

3   Properties of steady state equilibrium

The steady state equilibrium may or may not display sorting depending on the characteristics of the production function. In this section, we make the simplifying assumption that $\mu = \delta 0$. Proposition 1 states sufficient conditions for positive sorting to occur. First, define the worker type conditional CDF of firm types by,

$$\Omega(p|h) = \frac{\int_b^p g(h,p') dp'}{\int_b^{\bar{p}} g(h,p') dp'}. \tag{34}$$

One can then state the central characterization of sorting in steady state equilibrium.[2]

Proposition 1 *For any* $h \in [\underline{h}, \bar{h}]$, $\Omega(b|h) = 0$ *and* $\Omega(\bar{p}|h) = 1$. *Consider any pair* $(h_0, h_1) \in$
$[\underline{h}, \bar{h}] \times [\underline{h}, \bar{h}]$ *such that* $h_0 < h_1$. *If* $\kappa = 1$ *then for all* $p \in (b, \bar{p})$,

- $f_{hp}(h,p) > 0 \,\forall(h,p) \Rightarrow \Omega(p|h_0) > \Omega(p|h_1)$ *(supermodular).*

- $f_{hp}(h,p) < 0 \,\forall(h,p) \Rightarrow \Omega(p|h_0) < \Omega(p|h_1)$ *(submodular).*

- $f_{hp}(h,p) = 0 \,\forall(h,p) \Rightarrow \Omega(p|h_0) = \Omega(p|h_1)$ *(modular).*

The result generalizes to any $\kappa > 0$ *as long as* $R(h)$ *is weakly increasing (decreasing) in h when the production function is supermodular (submodular).*

---

[2]This proposition is given in Lentz (2007). We state it here for completeness.

Proof. See Lentz (2007). ∎

It is worth emphasizing that the stochastic dominance results in Proposition 1 do not cleanly extend to the firm productivity conditional worker skill distribution,

$$\Omega_p(h|p) = \frac{\int_{\underline{h}}^{h} g(h',p)dh'}{\int_{\underline{h}}^{\bar{h}} g(h',p)dh'}. \tag{35}$$

It is possible to locally break the stochastic dominance results for this conditioning.

4 Simulation

This section provides simulation results for the simplified model where $\nu(p) = 1\,\forall p$. The model parameters are set as follows,

$$\beta = 0.50$$

$$c_1 = 1.00$$

$$r = 0.05$$

$$\mu = 0.08$$

$$\delta_0 = 0.08$$

$$\delta_1 = 0.06$$

$$\lambda = 1.00$$

$$b = 0.01$$

$$\alpha = 0.50$$

$$m = 0.02$$

For any given choice of ρ, the search cost function and production function level parameters are set so as to obtain a steady state unemployment rate of $u = 0.05$ and an average wage of $w = 185$.

Figure 1 shows a comparison of allocation outcomes for three sorting cases; A supermodular production function ($\rho = -10$), a modular case ($\rho = 1$), and a submodular case ($\rho = 10$). The allocations are shown by comparing conditional averages of the joint match distribution $g(h,p)$.

It is seen that consistent with Proposition 1 the worker skill conditional firm productivity average $E[p|h]$ is monotonically increasing for the supermodular case, monotonically decreasing for

the submodular case, and flat for the modular case. The same pattern is broadly found for the reverse conditioning $E[h|p]$, but in this case, it is seen that the monotonicity is broken for the supermodular case.

Figure 2 presents simulated annual separation probabilities conditional on individual and firm wages. It is seen that the sorting properties of the model can affect the separation rate relationships profoundly. In both the modular and submodular cases, the relationships are decreasing, but in the supermodular case, it is seen that the individual wage conditional separation probability is non-monotonic and shows little variation.

Figure 3 presents the simulated relationships between value added per worker and the firm wage. In all cases, it is seen that higher productivity firms manage to extract a larger surplus share. In the submodular case, the firm wage estimate exceeds the value added per worker measure. This is purely a result of bias in the kernel regression. It must be in the model that wages are no greater than output.

A   Detailed derivations

Consider an employed worker of type h who is employed with a type p firm at employment contract (w, s). Denote by $q = q(h, w, p)$, the threshold type such that a meeting of an outside firm with type less than q has no impact on the worker's wage. Furthermore, adopt the short hand $V(h, q, p)$ as the value of employment to a type h worker who is employed with a type p firm subject to an employment contract set through bargaining where the worker had the threat point to accept outside employment with a type q firm. The value function, $\tilde{V}(h, w, p, s)$, for the employed worker is,

$$
\begin{aligned}
r\tilde{V}(h, p, w, s) &= w - c(s) + [\delta_1 + \Gamma(R(h))\delta_0\lambda(\theta)][V_0(h) - V(h, q, p)] + \\
&\quad s\lambda(\theta)\int_p^{\bar{p}}[V(h, p, p') - V(h, q, p)]\,d\Gamma(p') + \\
&\quad s\lambda(\theta)\int_q^p[V(h, p', p) - V(h, q, p)]\,d\Gamma(p') + \\
&\quad \delta_0\lambda(\theta)\int_{R(h)}^{\bar{p}}[V(h, R(h), p') - V(h, q, p)]\,d\Gamma(p') \\
&= w - c(s) + [\delta_1 + \Gamma(R(h))\delta_0\lambda(\theta)]V_0(h) - [\delta_0\lambda(\theta) + \delta_1 + s\lambda(\theta)(1 - \Gamma(q))]V(h, q, p) + \\
&\quad s\lambda(\theta)\int_p^{\bar{p}}[\beta V(h, p', p') + (1 - \beta)V(h, p, p)]\,d\Gamma(p') + \\
&\quad s\lambda(\theta)\int_q^p[\beta V(h, p, p) + (1 - \beta)V(h, p', p')]\,d\Gamma(p') + \\
&\quad \delta_0\lambda(\theta)\int_{R(h)}^{\bar{p}}[\beta V(h, p', p') + (1 - \beta)V_0(h)]\,d\Gamma(p')
\end{aligned}
$$

Integration by parts yields,

$$
\begin{aligned}
(r + \delta_0\lambda(\theta) + \delta_1)\tilde{V}(h, p, w, s) &= w - c(s) + [\delta_1 + \Gamma(R(h))\delta_0\lambda(\theta)]V_0(h) - s\lambda(\theta)(1 - \Gamma(q))V(h, q, p) + \\
&\quad s\lambda(\theta)(1 - \beta)(1 - \Gamma(p))V(h, p, p) + s\lambda(\theta)\beta(1 - \Gamma(p))V(h, p, p) + \\
&\quad s\lambda(\theta)\beta\int_p^{\bar{p}}(1 - \Gamma(p'))V_p'(h, p', p')\,dp' + \\
&\quad s\lambda(\theta)\beta(\Gamma(p) - \Gamma(q))V(h, p, p) - s\lambda(\theta)(1 - \beta)(1 - \Gamma(p))V(h, p, p) + \\
&\quad s\lambda(\theta)(1 - \beta)(1 - \Gamma(q))V(h, q, q) + s\lambda(\theta)(1 - \beta)\int_q^p(1 - \Gamma(p'))V'(h, p', p')\,dp' + \\
&\quad \delta_0\lambda(\theta)(1 - \beta)[1 - \Gamma(R(h))]V_0(h) + \delta_0\lambda(\theta)\beta[1 - \Gamma(R(h))]V_0(h) + \\
&\quad \delta_0\lambda(\theta)\beta\int_{R(h)}^{\bar{p}}[1 - \Gamma(p')]V'(h, p', p')\,dp'.
\end{aligned}
$$

By $V(h,q,p) = \beta V(h,p,p) + (1-\beta)V(h,q,q)$, one obtains.

$$
\begin{aligned}
(r + \delta_0\lambda(\theta) + \delta_1)\,\tilde{V}(h,p,w,s) = \ & f(h,p) - c(s) + (\delta_0\lambda(\theta) + \delta_1)\,V_0(h) + \\
& s\lambda(\theta)\beta \int_p^{\bar{p}} V'\left(h,p',p'\right)\left[1 - \Gamma(p')\right] dp' + \\
& s\lambda(\theta)(1-\beta) \int_q^p V'(h,p',p')[1 - \Gamma(p')]dp' + \\
& \delta_0\lambda(\theta)\beta \int_{R(h)}^{\bar{p}} V'\left(h,p',p'\right)\left[1 - \Gamma(p')\right] dp'.
\end{aligned}
\tag{36}
$$

By the envelope theorem it follows that,

$$
(r + \delta_0\lambda(\theta) + \delta_1)V_p'(h,p,p) = f_p'(h,p) - s(h,p)\lambda(\theta)\beta\big(1 - \Gamma(p)\big)V_p'(h,p,p)
$$

$$
\Updownarrow
$$

$$
V_p'(h,p,p) = \frac{f_p'(h,p)}{r + \delta_0\lambda(\theta) + \delta_1 + \beta s(h,p)\lambda(\theta)\big(1 - \Gamma(p)\big)}.
$$

Hence, equation (42) can be written as,

$$
\begin{aligned}
(r + \delta_0\lambda(\theta) + \delta_1)\,\tilde{V}(h,p,w,s) = \ & w - c(s) + (\delta_0\lambda(\theta) + \delta_1)\,V_0(h) + \\
& s\lambda(\theta)\beta \int_p^{\bar{p}} \frac{f_p'(h,p')[1 - \Gamma(p')]dp'}{r + \delta_0\lambda(\theta) + \delta_1 + \beta s(h,p')\lambda(\theta)[1 - \Gamma(p')]} + \\
& s\lambda(\theta)(1-\beta) \int_q^p \frac{f_p'(h,p')[1 - \Gamma(p')]dp'}{r + \delta_0\lambda(\theta) + \delta_1 + \beta s(h,p')\lambda(\theta)[1 - \Gamma(p')]} + \\
& \delta_0\lambda(\theta)\beta \int_{R(h)}^{\bar{p}} \frac{f_p'(h,p')[1 - \Gamma(p')]dp'}{r + \delta_0\lambda(\theta) + \delta_1 + \beta s(h,p')\lambda(\theta)[1 - \Gamma(p')]}.
\end{aligned}
$$

A.1   Steady state $G(h,q,p)$

The steady state condition on $G(h,q,p)$ is given by,

$$
\begin{aligned}
(1-u)\delta G(h,q,p) + (1-u)\lambda(\theta) \int_{\underline{h}}^h \int_{R(h')}^q \Big\{ & \big(1 - \Gamma(p)\big) \int_{q'}^q s\big(h',p'\big)dG\big(h',q',p'\big) \\
& + \big(1 - \Gamma(q)\big) \int_q^p s\big(h',p'\big)dG\big(h',q',p'\big) \Big\} = \\
\int_{\underline{h}}^h I(R(h') \le q)\big[\Gamma(p) - \Gamma(R(h'))\big]\lambda(\theta) & \Big[u[\delta_0 + \kappa s_0(h')]\upsilon(h') + \\
& (1-u)\delta_0 \int_{R(h')}^{\bar{p}} \int_{q'}^{\bar{p}} g(h',q',p')dp'dq'\Big] dh'.
\end{aligned}
\tag{37}
$$

Evaluate at $(h, \bar{p}, \bar{p})$ and differentiate with respect to h to obtain,

$$(\delta_0 \lambda(\theta) + \delta_1)(1-u) \int_{R(h)}^{\bar{p}} \int_{q'}^{\bar{p}} g(h, q', p') dp' dq' = [1 - \Gamma(R(h))]\lambda(\theta)\Big\{ u[\mu + \kappa s_0(h)]\upsilon(h') +$$
$$(1-u)\delta_0 \int_{R(h)}^{\bar{p}} \int_{q'}^{\bar{p}} g(h, q', p') dp' dq' \Big\}$$

$$\Updownarrow$$

$$(\delta_0 \lambda(\theta)\Gamma(R(h)) + \delta_1)(1-u) \int_{R(h)}^{\bar{p}} \int_{q'}^{\bar{p}} g(h, q', p') dp' dq' = u[1 - \Gamma(R(h))]\lambda(\theta)[\mu + \kappa s_0(h)]\upsilon(h)$$

$$\Updownarrow$$

$$\delta_0(1-u) \int_{R(h)}^{\bar{p}} \int_{q'}^{\bar{p}} g(h, q', p') dp' dq' = \frac{\delta_0 \lambda(\theta)[1 - \Gamma(R(h))]}{\delta_0 \lambda(\theta)\Gamma(R(h)) + \delta_1} u[\mu + \kappa s_0(h)]\upsilon(h) \tag{38}$$

Insert this into equation (37),

$$\frac{\delta_0 \lambda(\theta) + \delta_1}{\lambda(\theta)} G(h, q, p) + \int_{\underline{h}}^{h} \int_{R(h')}^{q} \Big[[1 - \Gamma(p)] \int_{q'}^{q} s(h', p') dG(h', q', p')$$
$$+ [1 - \Gamma(q)] \int_{q}^{p} s(h', p') dG(h', q', p') \Big] =$$
$$\frac{u}{1-u} \int_{\underline{h}}^{h} I(R(h') \le p)[\Gamma(p) - \Gamma(R(h'))][\mu + \kappa s_0(h')]\upsilon(h') \frac{\delta_1 + \delta_0 \lambda(\theta)}{\delta_0 \lambda(\theta)\Gamma(R(h)) + \delta_1} dh'. \tag{39}$$

Evaluate (39) at $(\bar{h}, \bar{p}, \bar{p})$ to obtain,

$$\frac{\delta_0 \lambda(\theta) + \delta_1}{\lambda(\theta)} = \frac{u}{1-u} \int_{\underline{h}}^{\bar{h}} [1 - \Gamma(R(h'))] \frac{\delta_1 + \delta_0 \lambda(\theta)}{\delta_0 \lambda(\theta)\Gamma(R(h)) + \delta_1} [\mu + \kappa s_0(h')]\upsilon(h') dh'$$

$$\Updownarrow$$

$$\frac{u}{1-u} = \left[\int_{\underline{h}}^{\bar{h}} \frac{[1 - \Gamma(R(h'))][\mu + \kappa s_0(h')]}{\delta_0 \Gamma(R(h')) + \delta_1/\lambda(\theta)} \upsilon(h') dh' \right]^{-1}$$

$$\Updownarrow$$

$$u = \left[\int_{\underline{h}}^{\bar{h}} \left(1 + \frac{[1 - \Gamma(R(h'))][\mu + \kappa s_0(h')]}{\delta_0 \Gamma(R(h')) + \delta_1/\lambda(\theta)} \right) d\Upsilon(h') \right]^{-1}.$$

One then obtains,

$$\int_{\underline{h}}^{h} \int_{R(h')}^{q} \left[\int_{q'}^{q} [\delta/\lambda(\theta) + [1 - \Gamma(p)]s(h', p')]g(h', q', p') dp' \right.$$
$$\left. + \int_{q}^{p} [\delta/\lambda(\theta) + [1 - \Gamma(q)]s(h', p')]g(h', q', p') dp' \right] dq' dh' =$$
$$\frac{\delta}{\lambda(\theta)} \frac{\int_{\underline{h}}^{h} I(R(h') \le q)[\Gamma(p) - \Gamma(R(h'))] \frac{\mu + \kappa s_0(h')}{\delta_0 \Gamma(R(h')) + \delta_1/\lambda(\theta)} d\Psi(h')}{\int_{\underline{h}}^{\bar{h}} \frac{[1 - \Gamma(R(h'))][\mu + \kappa s_0(h')]}{\delta_0 \Gamma(R(h')) + \delta_1/\lambda(\theta)} d\Psi(h')}. \tag{40}$$

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A.2 Steady state equilibrium solution for $\Upsilon(h)$

Consider the equilibrium condition,

$$\Psi(h) = u\Upsilon(h) + (1-u)\,G(h,\bar{p}).$$

Differentiate with respect to h to obtain,

$$
\begin{aligned}
\psi(h) &= uv(h) + (1-u)\int_b^{\bar{p}} g(h,p')\,dp'\\
&= \left[1 + \frac{[1-\Gamma(R(h))][\mu + \kappa s_0(h)]}{\delta_0\Gamma(R(h)) + \delta_1/\lambda(\theta)}\right] uv(h),
\end{aligned}
$$

where the last equality follows from equation (38). By the steady state unemployment rate expression in equation (??), it follows that,

$$
\psi(h) = \frac{\left[1 + \frac{[1-\Gamma(R(h))][\mu + \kappa s_0(h)]}{\delta_0\Gamma(R(h)) + \delta_1/\lambda(\theta)}\right] v(h)}{\int_{\underline{h}}^{\bar{h}}\left(1 + \frac{[1-\Gamma(R(h'))][\mu + \kappa s_0(h')]}{\delta_0\Gamma(R(h')) + \delta_1/\lambda(\theta)}\right) v(h')\,dh'}, \tag{41}
$$

which is an integral equation for $\Upsilon(h)$ as a function of $\Psi(h)$. Define,

$$\Delta(h) = \frac{[1-\Gamma(R(h))][\mu + \kappa s_0(h)]}{\delta_0\Gamma(R(h)) + \delta_1/\lambda(\theta)}.$$

Then restate equation (41),

$$v(h) = \left[1 + \int_{\underline{h}}^{\bar{h}} \Delta(h')\,v(h')\,dh'\right]\frac{\psi(h)}{1+\Delta(h)}.$$

Use equation (41) to solve for $1 + \int_{\underline{h}}^{\bar{h}}\Delta(h')\,v(h')\,dh'$. First, some minor manipulation,

$$\psi(h) + \psi(h)\int_{\underline{h}}^{\bar{h}} \Delta(h')\,v(h')\,dh' = [1+\Delta(h)]\,v(h)$$

$$\Updownarrow$$

$$v(h) - \frac{\psi(h)}{1+\Delta(h)}\int_{\underline{h}}^{\bar{h}} \Delta(h')\,v(h')\,dh' = \frac{\psi(h)}{1+\Delta(h)}$$

$$\Updownarrow$$

$$\Delta(h)\,v(h) - \frac{\psi(h)\,\Delta(h)}{1+\Delta(h)}\int_{\underline{h}}^{\bar{h}} \Delta(h')\,v(h')\,dh' = \frac{\psi(h)\,\Delta(h)}{1+\Delta(h)}.$$

Now, integrate from $\underline{h}$ to $\bar{h}$ to obtain,

$$\int_{\underline{h}}^{\bar{h}} \Delta\left(h'\right) \upsilon\left(h'\right) dh' \left[1 - \int_{\underline{h}}^{\bar{h}} \frac{\psi\left(h'\right)\Delta\left(h'\right)}{1+\Delta\left(h'\right)} dh'\right] = \int_{\underline{h}}^{\bar{h}} \frac{\psi\left(h'\right)\Delta\left(h'\right)}{1+\Delta\left(h'\right)} dh'$$

$$\Updownarrow$$

$$\begin{aligned}
1 + \int_{\underline{h}}^{\bar{h}} \Delta\left(h'\right)\upsilon\left(h'\right) dh' &= 1 + \frac{\int_{\underline{h}}^{\bar{h}} \frac{\psi(h')\Delta(h')}{1+\Delta(h')} dh'}{1 - \int_{\underline{h}}^{\bar{h}} \frac{\psi(h')\Delta(h')}{1+\Delta(h')} dh'} \\
&= \frac{1}{1 - \int_{\underline{h}}^{\bar{h}} \frac{\Delta(h')}{1+\Delta(h')}\psi\left(h'\right) dh'} \\
&= \frac{1}{\int_{\underline{h}}^{\bar{h}} \left[1 - \frac{\Delta(h')}{1+\Delta(h')}\right]\psi\left(h'\right) dh'} \\
&= \frac{1}{\int_{\underline{h}}^{\bar{h}} \frac{1}{1+\Delta(h')}\psi\left(h'\right) dh'}.
\end{aligned}$$

Hence, one obtains the solution,

$$\upsilon\left(h\right) = \frac{\left[1+\Delta\left(h\right)\right]^{-1}\psi\left(h\right)}{\int_{\underline{h}}^{\bar{h}} \left[1+\Delta\left(h'\right)\right]^{-1}\psi\left(h'\right) dh'},$$

which can also be written as,

$$\Upsilon\left(h\right) = \frac{\int_{\underline{h}}^{h} \frac{\delta_0 \Gamma(R(h'))+\delta_1/\lambda(\theta)}{\delta_0 \Gamma(R(h'))+\delta_1/\lambda(\theta)+[1-\Gamma(R(h'))][\mu+\kappa s_0(h')]} d\Psi\left(h'\right)}{\int_{\underline{h}}^{\bar{h}} \frac{\delta_0 \Gamma(R(h'))+\delta_1/\lambda(\theta)}{\delta_0 \Gamma(R(h'))+\delta_1/\lambda(\theta)+[1-\Gamma(R(h'))][\mu+\kappa s_0(h')]} d\Psi\left(h'\right)}.$$

B Firm labor force composition is independent of firm size

Consider a labor force that consists of k types. For the purpose of this argument, a type i worker is characterized by a hire rate h_i and a separation rate d_i. Firm entry and exit takes place through the zero labor force size pool. Each worker i size process is independent. Hence, the distribution of the number of type i workers employed by the firm will be Poisson distributed,

$$m_n^i = \frac{\left(\frac{h_i}{d_i}\right)^n \exp\left(-\frac{h_i}{d_i}\right)}{n!}.$$

Denote by $\vec{n} = (n_1, n_2, \ldots, n_k)$ the composition of the firm's labor force. The mass of size n firms is formed based on the sum of the individual worker type distributions,

$$\begin{aligned}
m_n &= \sum_{\{\vec{n}\geq 0 | \sum n_i = n\}} \prod_{i=1}^{k} m_{n_i}^i \\
&= \frac{\left[\sum_{i=1}^{k} \frac{h_i}{d_i}\right]^n \exp\left(-\sum_{i=1}^{k} \frac{h_i}{d_i}\right)}{n!},
\end{aligned}$$

which is just a Poisson in the sum of the individual hiring and separation rate fraction. Consider the expectation of the share of type i workers in the firm's labor force conditional on the firm having n workers,

$$
\begin{aligned}
E\left[\frac{n_i}{n}|n\right] &= \frac{\sum_{\{\vec{n}\geq 0|\sum n_j = n\}} \frac{n_i}{n} \prod_{j=1}^{k} m_{n_j}^{j}}{m_n} \\
&= \frac{\sum_{\{\vec{n}\geq 0|\sum n_j = n\}} n! \frac{n_i}{n} \frac{\prod_{j=1}^{k}\left(\frac{\eta_j}{\delta_j}\right)^{n_j}}{\prod_{j=1}^{k} n_j!}}{\sum_{\{\vec{n}\geq 0|\sum n_j = n\}} n! \frac{\prod_{j=1}^{k}\left(\frac{\eta_j}{\delta_j}\right)^{n_j}}{\prod_{j=1}^{k} n_j!}} \\
&= \frac{\left(\frac{\eta_i}{\delta_i}\right)\left[\sum_{i=1}^{k}\frac{h_i}{d_i}\right]^{n-1}}{\left[\sum_{i=1}^{k}\frac{h_i}{d_i}\right]^{n}} \\
&= \frac{\frac{h_i}{d_i}}{\sum_{i=1}^{k}\frac{h_i}{d_i}}
\end{aligned}
$$

where the second to last step applies the multinomial theorem. Hence, the share of type i workers in the firm's labor force is independent of the size of the firm's labor force. Consequently, the firm's overall worker separation rate is not size dependent.

C   Employment contract bargaining

At the beginning of an employment relationship, the firm and the worker bargain over a constant wage and worker's search intensity that will remain in effect until the relationship terminates or both parties consent to renegotiation. The bargaining game is an application of the alternating offers game of Rubinstein (1982) and most resembles the exogenous break down version as presented in Binmore, Rubinstein, and Wolinsky (1986). The following two subsections present the subgame perfect equilibrium for the case of an unemployed worker worker and a worker who is renegotiating subsequent to an outside offer, respectively. The arguments are closely related to the bargaining games described in Cahuc, Postel-Vinay, and Robin (2006), although the bargaining is simplified to take place in artificial time with zero disagreement values and the possibility of meeting another employer during bargaining is eliminated.

The outcomes of the alternating offers games are identical to that of axiomatic Nash bargaining where the threat point of the firm is always zero for the firm, and the worker's threat point is either

unemployment or full surplus extraction from the least productive of the two firms competing over the worker. This is the argument presented in Dey and Flinn (2005). Specifically, the bargaining outcome of an unemployed worker maximizes the Nash product,

$$\{w_0(h,p), s(h,p)\} = \arg\max_{w,s} \left(\tilde{V}(h,p,w,s) - V_0(h)\right)^{\beta} \tilde{J}(h,w,p,s)^{(1-\beta)}, \tag{42}$$

which yields the worker valuation,

$$V(h, R(h), p) = \beta V(h,p,p) + (1-\beta)V_0(h). \tag{43}$$

The inclusion of the reservation productivity argument implicitly states that the worker will only accept to bargain with employer types greater than $R(h)$.

The outcome of a worker bargaining with two employer types, q and p such that $p > q$ is that the worker will negotiate an employment contract with the type p firm with a threat point of full surplus extraction and efficient search intensity with the lower type firm, $V(h,q,q)$. Hence, the employment contract that results from this bargaining setting is,

$$\{w(h,q,p), s(h,p)\} = \arg\max_{w,s} \left(\tilde{V}(h,p,w,s) - V(h,q,q)\right)^{\beta} \tilde{J}(h,w,p,s)^{(1-\beta)}. \tag{44}$$

The bargaining outcome is,

$$V(h,q,p) = \beta V(h,p,p) + (1-\beta)V(h,q,q). \tag{45}$$

In both cases (42) and (44), the agreed upon search intensity $s(h,p)$ is the one that maximizes total match surplus. This is the jointly efficient search intensity level and does not depend on the specific surplus split dictated by bargaining power and threat points.

C.1 Unemployed worker

Consider an alternating offers game where the worker makes an offer (w_e, s_e) to the firm. If the firm accepts, employment starts and the worker receives payoff $\tilde{V}(h,p,w_e,s_e)$ and the firm receives $\tilde{J}(h,p,w_e,s_e) = \tilde{V}(h,p,f(h,p),s_e) - \tilde{V}(h,p,w_e,s_e)$. If the firm rejects the offer, the bargaining breaks down with exogenous probability Δ. If so, the firm receives a zero payoff and the worker goes back to unemployment and receives $V_0(h)$. If bargaining does not break

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down, the bargaining moves to the next round where the firm makes an offer (w_f, s_f) with probability $1 - \beta$ and the worker gets to make the offer (w_e, s_e) with probability β. If the firm makes the offer and the worker accepts, the worker receives $\tilde{V}(h, p, w_f, s_f)$ and the firm receives $\tilde{J}(h, p, w_f, s_f) = \tilde{V}(h, p, f(h, p), s_f) - \tilde{V}(h, p, w_f, s_f)$. If the worker rejects, the game moves on to the next round if no break down occurs. And again, the worker will make the offer with probability β and the firm with probability $1 - \beta$. The game continues like this ad infinitum or until agreement is reached. Disagreement payoffs are zero and the discount rate between rounds is zero.

Both the worker and the firm will offer the same search intensity, $s_e = s_f = s(h, p)$, where $s(h, p) = \arg\max_s \tilde{V}(h, p, f(h, p), s)$. Furthermore, consider the strategies where the worker accepts any offer (w, s) such that $\tilde{V}(h, p, w, s) \geq \tilde{V}(h, p, w_f, s(h, p))$ and rejects any offer such that $\tilde{V}(h, p, w, s) < \tilde{V}(h, p, w_f, s(h, p))$. Similarly, the firm accepts any offer (w, s) such that $\tilde{J}(h, p, w, s) \geq \tilde{J}(h, p, w_e, s(h, p))$ and rejects any offer such that $\tilde{J}(h, p, w, s) < \tilde{J}(h, p, w_e, s(h, p))$.

By definition the firm's payoff satisfies $\tilde{J}(h, p, w, s) = \tilde{V}(h, p, f(h, p), s) - \tilde{V}(h, p, w, s)$. Hence, a firm accepts any offer such that

$$\tilde{V}(h, p, w, s) \leq \tilde{V}(h, p, w_e, s(h, p)) - \tilde{V}(h, p, f(h, p), s(h, p)) + \tilde{V}(h, p, f(h, p), s). \qquad (46)$$

It is seen that the right hand side of the firm acceptance condition (46) is maximized for $s = s(h, p)$ and does not depend on w. Hence, any worker deviation $s'_e \neq s_e = s(h, p)$ that will be accepted by the firm must result in a worker payoff $\tilde{V}(h, p, w, s'_e) < \tilde{V}(h, p, w_e, s(h, p))$, for any w, which is not profitable.

A similar argument can be made that the firm will not want to deviate from $s_f = s(h, p)$. The worker will accept any offer such that,

$$\tilde{J}(h, p, w, s) \leq \tilde{V}(h, p, f(h, p), s) - \tilde{V}(h, p, w_f, s(h, p)). \qquad (47)$$

It is seen that the right hand side of the worker acceptance decision (47) is maximized for $s = s(h, p)$ and that it does not depend on w. Hence, any firm deviation $s'_f \neq s_f = s(h, p)$ that will be accepted by the worker must result in a firm payoff $\tilde{J}(h, p, w, s'_f) < \tilde{J}(h, p, w_f, s_f)$, for any w, which is not profitable.

It also follows directly from the above acceptance arguments that any strategy that prescribes $s_e \neq s(h, p)$ or $s_f \neq s(h, p)$ cannot be an equilibrium because a deviation to $s(h, p)$ will be profitable.

Now consider potential deviations in the wage. The worker's payoff $\tilde{V}(h, p, w, s_e)$ is monotonically increasing in w. It follows directly from (46) that any worker wage offer deviation w'_e that will be accepted by the firm is such that $w'_e \leq w_e$. This is not profitable. Any other deviation will not be accepted by the firm and is therefore also not profitable. A similar argument applies to possible firm wage offer deviations.

Sub game perfection of the acceptance strategies requires that the worker is indifferent between accepting the firm's offer (w_f, s_f) and rejecting it. A similar indifference applies on the firm side. This disciplines the acceptance levels by,

$$\hat{V}(w_f) = (1 - \Delta)\left[\beta\hat{V}(w_e) + (1 - \beta)\hat{V}(w_f)\right] + \Delta V_0(h) \tag{48}$$

$$\hat{J}(w_e) = (1 - \Delta)\left[\beta\hat{J}(w_e) + (1 - \beta)\hat{J}(w_f)\right] \tag{49}$$

where $\hat{V}(w) = \tilde{V}(h, p, w, s(h, p))$ and $\hat{J}(w) = \tilde{V}(h, p, w, s(h, p))$. Equations (48) and (49) can be rewritten as,

$$\beta\left[\hat{V}(w_f) - \hat{V}(w_e)\right] = \Delta\left[V_0(h) - \beta\hat{V}(w_e) - (1 - \beta)\hat{V}(w_f)\right] \tag{50}$$

$$(1 - \beta)\left[\hat{J}(w_f) - \hat{J}(w_e)\right] = \Delta\left[\beta\hat{J}(w_e) + (1 - \beta)\hat{J}(w_f)\right]. \tag{51}$$

Taking the limit as $\Delta \to 0$, equations (48) and (49) imply that $w_f \to w_e$. Denote the common limit by w. Hence,

$$\frac{\partial\hat{V}(w)}{\partial w} = \lim_{\Delta \to 0} \frac{\hat{V}(w_f) - \hat{V}(w_e)}{w_f - w_e}$$

$$\frac{\partial\hat{J}(w)}{\partial w} = \lim_{\Delta \to 0} \frac{\hat{J}(w_f) - \hat{J}(w_e)}{w_f - w_e}.$$

Since changes in w only affect the match surplus split, it follows that $\partial\hat{V}(w)/\partial w = -\partial\hat{J}(w)/\partial w$. Hence, taking the limit $\Delta \to 0$ in equations (50) and (51) yields,

$$-\frac{\beta}{1 - \beta} = \frac{V_0(h) - \beta\hat{V}(w) - (1 - \beta)\hat{V}(w)}{\beta\hat{J}(w) + (1 - \beta)\hat{J}(w)}$$

$$\Updownarrow$$

$$\hat{V}(w) = \beta\hat{V}(f(h, p)) + (1 - \beta)V_0(h). \tag{52}$$

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Hence, as the break down probability goes to zero, the outcome of the alternating offers game limits to the outcome of the axiomatic Nash bargaining outcome in equation (43).

C.2 Employed worker

Cahuc, Postel-Vinay, and Robin (2006) provide a strategic bargaining foundation for the axiomatic Nash bargaining outcome in equation (45). The outcome is a subgame perfect equilibrium in a game based on firms submitting bids for the worker subject to a worker's option to use the bids as threat points in a subsequent strategic bargaining game. In the game between two employers of types q and p, respectively, where $q \leq p$, the higher type firm wins by submitting a contract bid $(w, s(h, p))$ as stated in equation (45).

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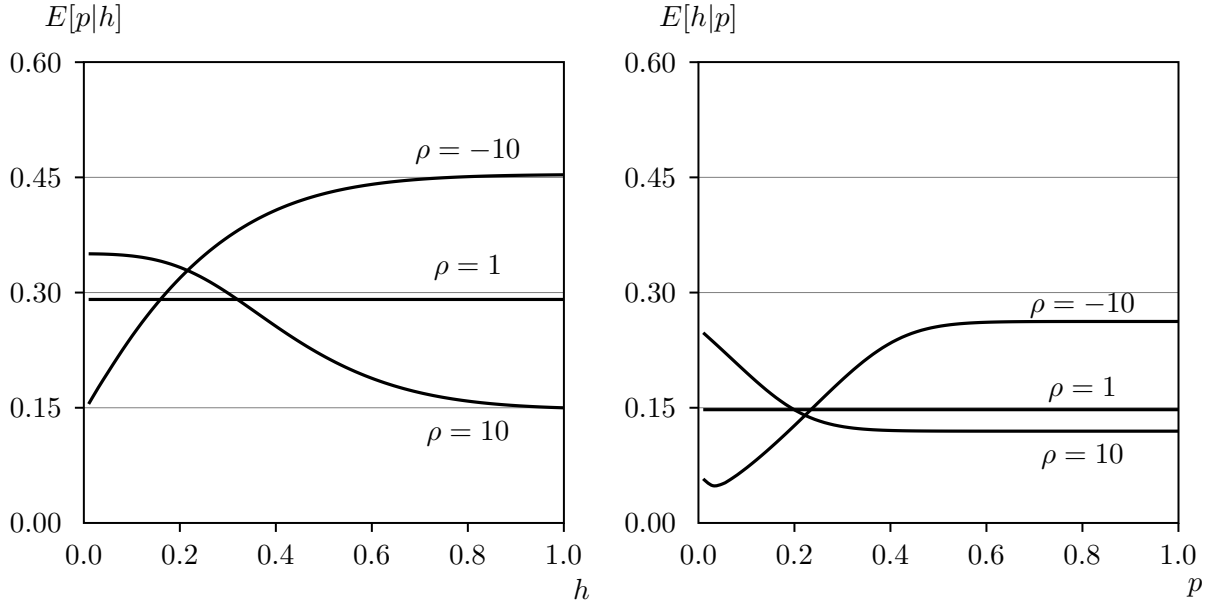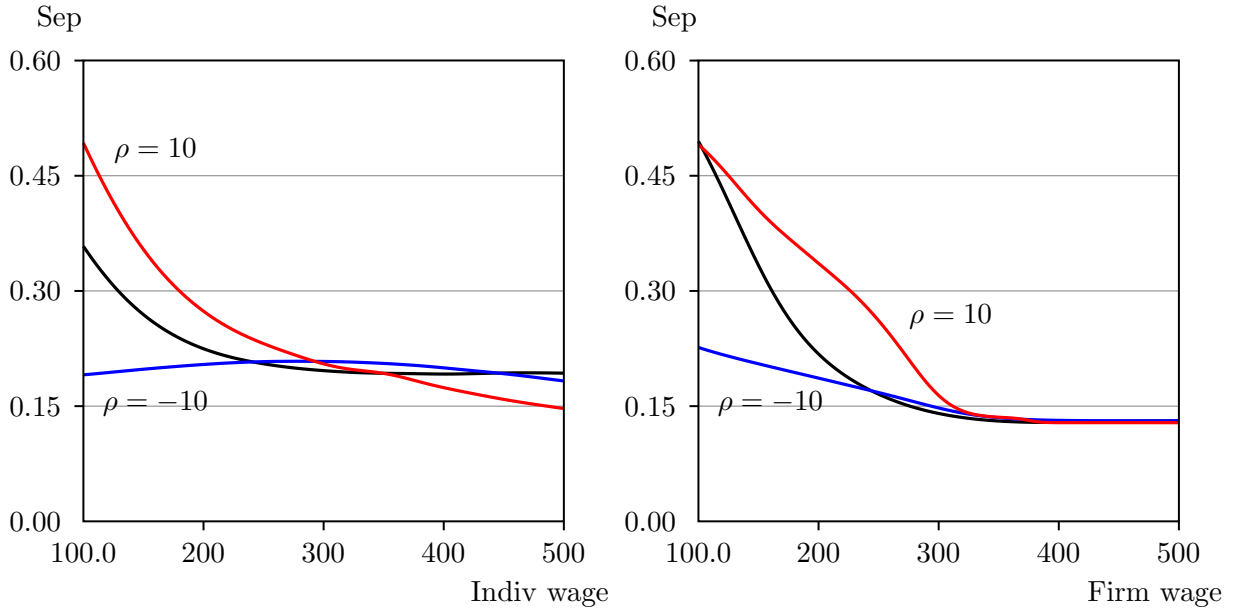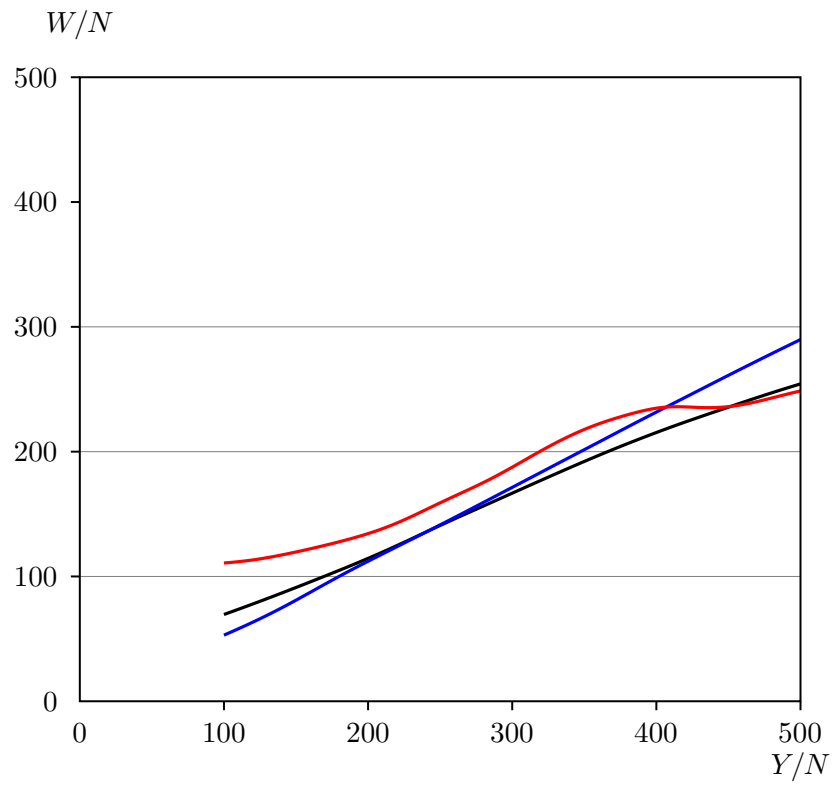
Figure 1: Conditional expectations

Figure 2: Wage conditional annual separation probabilities

Note: The blue line represents the supermodular case ($\rho = -10$), the red line is the submodular case ($\rho = 10$), and the black line is the modular case ($\rho = 1$).

Figure 3: Productivity and wages

Note: The blue line represents the supermodular case ($\rho = -10$), the red line is the submodular case ($\rho = 10$), and the black line is the modular case ($\rho = 1$).