Solar Geoengineering, Uncertainty, and the Social Cost of Carbon

Abstract

We consider the socially optimal use of geoengineering as a tool to manage climate change. Geoengineering offers the possibility of reducing the damages from atmospheric greenhouse gas concentrations, potentially at a lower cost than reducing emissions. If so, then an optimal policy path can include less abatement than is recommended by models that do not include geoengineering, and the price of carbon will be lower. Solar geoengineering reduces temperature but does not reduce atmospheric or ocean carbon concentrations, and that carbon may cause damages apart from temperature rise. Finally, uncertainty about both climate change and about geoengineering affects the optimal deployment of geoengineering. We explore these issues with both an analytical model and a numerical simulation. The optimal carbon tax is lower than the tax recommended by the model without geoengineering, substantially so depending on the parameterizations of geoengineering costs and benefits. Carbon concentrations are higher but temperature changes are lower when allowing for geoengineering. All policy paths are sensitive to calibrated parameter values, and the optimal level of geoengineering is more sensitive to climate uncertainty than is the optimal level of abatement. The point estimates should be interpreted with caution since there is a great deal of uncertainty surrounding feasibility and side effects of geoengineering.

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Garth Heutel
Department of Economics
Georgia State University
and NBER
gheutel@gsu.edu

Juan Moreno-Cruz
School of Economics
Georgia Institute of Technology
morenocruz@gatech.edu

Soheil Shayegh
Department of Global Ecology
Carnegie Institution of Washington, Stanford.
sshayegh@carnegiescience.edu

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Geoengineering is "the poor orphan that almost nobody wants to mention." (Schelling, 2007, p. 348)
"Most economic analyses of climate change, however, have ignored geoengineering." (Barrett, 2008, p. 46)
"I suspect, however, that there is at least a small chance we would live to regret treating the whole subject [geoengineering] as taboo." (Summers, 2007, p. xxiv)

I. Introduction

Greenhouse gases (GHGs) like carbon dioxide (CO_2) contribute to climate change and thus create negative externalities. The standard Pigouvian solution to the market failure caused by negative externalities is to price the externality at marginal external damages so that producers internalize the externality and produce at the optimal quantity. The Pigouvian tax is set at marginal external damages at the optimal quantity. Geoengineering (GE, also called climate engineering or climate intervention) is an alternative way to reduce the damages from GHGs: instead of reducing the quantity of GHGs, GE can, at leas tin part, reduce the damages that they inflict. Because GE is not aimed at reducing emissions, a Pigouvian tax on GHG emissions will do nothing to create incentives for GE. If GE is part of the optimal policy portfolio, then a Pigouvian tax alone cannot bring about the first-best. Furthermore, if the Pigouvian tax is set at the level of marginal external damages without GE, then the tax will be too high relative to the optimum level (which is equal to marginal external damages with GE) and the level of abatement and abatement expenditures will be too high, resulting in welfare loss. It has been argued that the implementation of GE may be substantially cheaper than abatement, possibly creating a very large welfare loss from ignoring GE.[1] But GE also introduces new sources of damages and uncertainty, possibly eroding on the welfare gains from its implementation, even possibly increasing the welfare loss from ignoring it.

[1] For example, McClellan et al. (2012) find that GE could be undertaken globally for as little as \$1-\$3 billion annually. By comparison, the EPA's cost estimate of its proposed Clean Power Plan puts the annual cost at around \$4-\$8 billion annually, and that policy only yields a 30% reduction in CO_2 emissions just from power plants in the United States.

The purpose of this paper is to investigate, theoretically and numerically, how the possibility of GE affects optimal climate policy. Does GE substantially reduce the optimal carbon tax? Does ignoring GE lead to policies that encourage too much abatement at too high a cost? How does uncertainty over climate change and over geoengineering damages affect optimal policy? How important is the fact the GE reduces temperature but does not reduce carbon concentrations? We develop a theoretical model that captures these effects and demonstrates the welfare effects of introducing or ignoring GE. We augment a standard integrated assessment model (IAM) of climate change by adding the possibility of a specific type of GE to the Dynamic Integrated Climate-Economy (DICE) model (Nordhaus 2008). Costs and benefits of GE are calibrated from various prior estimates, though we note that there is substantial uncertainty. Therefore, we also explore conditions under which GE will or will not represent a substantial component of optimal policy. We caution that the purpose of this paper is not to argue one way or the other about the merits of GE or to estimate the optimal level of its deployment, but rather to investigate the qualitative point that ignoring GE in models may lead to biased and incomplete policy recommendations.

There is a growing literature that examines the economics of geoengineering.[2] Barrett (2008) explores the "incredible economics of geoengineering," by which he means the fact that GE is (potentially) so much cheaper than emissions abatement that it could be undertaken by a single country.[3] This creates a unique set of administrative problems. In fact, if the key problem with administering abatement policy is the inability to achieve international consensus to act, the key problem with GE might be ensuring that there is not too *much* implemented, since any number of

[2] In addition to the small but growing literature in economics on GE, there is a large scientific literature on the subject. Latham et al. (2014a) and the associated special journal issue provide a recent introduction.

[3] The prospect of low-cost GE is not universally accepted. For instance, Keller et al. (2014) use an Earth system model to simulate several different types of GE in the presence of high GHG emissions (no abatement), and they find that the effects of GE on warming are limited (less than an 8% reduction) and the side effects are potentially severe.

nations may do it independently. A series of studies examine this issue of international governance for GE. Ricke et al. (2013) look at the incentives behind the formation of coalitions to implement GE. These incentives are different than those behind coalitions to abate GHGs. With GE, there are incentives to keep coalitions small so that action can be taken. Victor (2008) argues for norms to govern the deployment of GE. Weitzman (2012) models GE as a "free-driver" problem analogous to the "free-rider" problem of abatement. He notes that GE, depending on the level at which it is undertaken and the nation in question, can be either a public good or a public bad (thus he labels it a "public gob"). Moreno-Cruz (2011) models two countries agreeing on both mitigation and geoengineering, and shows that GE can lead to inefficiently high levels of mitigation. These papers are primarily concerned with how international agreements can be crafted to implement GE (or to prevent too much implementation).[4]

By contrast, a perhaps more fundamental question (though one less-studied) is how much GE is optimal? Moreno-Cruz and Keith (2013) incorporate GE into a two-period model of climate change and solve for optimal policy. They find that the uncertainty related to GE is an important determinant of optimal policy. Including GE can reduce the overall costs of climate policy by around 2 percentage points of GDP. In this case, abatement and GE are substitute policy levers. Other papers have added GE to integrated assessment models (IAM) and examined the policy implications. Bickel and Lane (2009) show that GE promises potentially large net benefits, though there is substantial uncertainty. They model two types of GE: solar radiation management (SRM) and air capture (AC), and they conclude that SRM is cheaper and more cost-effective. They conduct a benefit-cost analysis of various levels of implementation of GE, and they consider how implementing GE affects carbon taxes. But, they do not solve for an optimal level of GE. Goes et al. (2011) make several modifications to the DICE model,

[4] Barrett (2014) discusses the literature on the governance of geoengineering, what he calls "the fundamental problem posed by geoengineering." Rayner et al. (2013) present the "Oxford Principles," a set of five guidelines for international GE governance. Lloyd and Oppenheimer (2014) argue for an international agreement with a small number of nations. A similar argument is related to moral hazard: deployment of GE may reduce or eliminate the willingness to reduce carbon emissions (Corner and Pidgeon 2014).

including allowing GE and refining the climate dynamics. Their specification imposes an exogenous intermittency in GE which makes it less effective.[5] They present summaries of policies with an optimal mix of abatement and GE (subject to the intermittency), but they do not present implications for policy, i.e. carbon taxes with GE. Bickel and Agrawal (2013) extend the analysis of Goes et al. (2011) by considering alternate conditions under which GE would be deployed; in contrast to Goes et al. (2011), Bickel and Agrawal (2013) find that under some scenarios a substitution of GE for abatement can pass a cost-benefit test. Gramstad and Tjøtta (2010) include GE in DICE and conduct a cost-benefit analysis of GE under various assumptions about the level undertaken and its costs. Under all specifications, GE passes a cost-benefit analysis, with net benefits ranging from $1.5 trillion to $17.8 trillion. Postponement of GE by 30-50 years reduces those net benefits by less than 10%. They do not consider carbon taxes or the optimal level of GE. [6]

The contribution of our paper relative to this other literature on the policy implications of GE is threefold. First, we focus on how the inclusion of GE affects optimal abatement and the optimal carbon tax. Our theoretical model shows that including GE reduces the optimal carbon tax. Since GE appears to be so much cheaper than abatement, it is possible that including GE will drastically reduce the optimal carbon tax. It is theoretically possible that optimal policy will involve a corner solution with no abatement (and hence no carbon taxes), though we show numerically that this does not hold because of GE's inability to deal with carbon concentrations. To quantify this, we modify the DICE model to include the possibility of GE and use it to solve for optimal policy, where both abatement and GE are policy choices. While solving for the optimal carbon price is admittedly a straightforward extension of other papers that have used an IAM with GE to find optimal policy, we argue that it is nonetheless an

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[5] Jones et al. (2013) also investigate the effect of abrupt suspension of GE (a "termination effect"), using a simulation of 11 different climate models. Also see Ross and Matthews (2009).

[6] Klepper and Rickels (2012 and 2014) provide review articles on the economics of geoengineering. Emmerling and Tavoni (2013) use a different IAM, WITCH, to model GE and abatement policy.

important contribution that should not be overlooked. If GE means that the optimal carbon price is much lower than current estimates of the social cost of carbon, this has very important policy implications. We calculate the welfare loss of ignoring this fact. We also explore how different assumptions about various parameter values affect the time path of optimal policy. For instance, it is well known that small changes to the discount rate used in the model can have large changes in optimal abatement paths, but little is known about how discounting affects optimal GE paths, or how discounting affects abatement when GE is an option. Should GE be delayed, or implemented immediately? Should GE ramp up, like a carbon price does in DICE, or start high and taper? [7]

Our second contribution to the literature is our focus on uncertainty. There are substantial uncertainties about the costs, benefits, and risks of GE given the present state of scientific understanding.[8] There is also uncertainty in our understanding of the climate, in particular over the climate sensitivity parameter, which captures by how much temperature changes due to a doubling of CO_2 concentrations. We characterize these uncertainties in our analytical model and derive policy implications. We also use a stochastic version of DICE to model uncertainty in geoengineering and in the climate system.

Third, in our models (analytical and numerical) we explicitly distinguish between damages from carbon concentrations and damages from temperature. Unlike mitigation or abatement, solar geoengineering reduces temperatures without reducing carbon concentrations, either atmospheric or oceanic. Both types of carbon stocks may lead to damages, even if temperatures are brought back to preindustrial levels. For instance, ocean acidification may deplete corals and fisheries, and atmospheric carbon may affect precipitation patterns. Other papers have mentioned this issue, but to our knowledge

[7] Barrett (2014) considers four different options for the time path of GE, and Keith (2013) recommends starting at a low level of GE and gradually increasing its use, but neither uses an IAM to generate optimal policy.
[8] The National Academy of Sciences has recently issued a report providing a technical evaluation of GE proposals, jointly sponsored by the NOAA, the CIA, NASA, and the Energy Department (National Research Council 2015).

ours is the first to incorporate it into a theoretical model or numerical simulation of geoengineering policy.

We find that GE unambiguously lowers the optimal level of abatement and the optimal price of carbon in the model. The degree to which it does so is sensitive to parameter values. In our base case specification, the optimal level of abatement is up to 25 percentage points lower than the optimal level without GE, and the elimination of all carbon emissions is delayed by five decades. Ignoring GE can increase overall costs of climate change by one-half to one percent of GDP. Optimal abatement levels are less sensitive to parameter values and to uncertainty in climate sensitivity than are optimal GE levels. The degree to which damages from climate change arise from carbon directly, rather than from temperature, substantially affects optimal GE deployment; if a high fraction of damages are from carbon, then GE is used less intensively.

The next section of the paper introduces our theoretical model, which provides the framework for our inclusion of GE into the DICE model. Section 3 describes how we include GE in the DICE model and how it is calibrated. Section 4 presents our simulation results, and section 5 concludes.

II. Theoretical Model

Consider a representative agent who has access to an endowment of a fixed stock of capital k. That capital can be allocated in three ways: towards production k_p, towards abatement k_a, or towards geoengineering k_g. The resource constraint is thus $k = k_p + k_a + k_g$. Allocating capital towards production yields a level of potential output $f\left(k_p\right)$, with $f' > 0$ and $f'' < 0$. This is potential output, because actual output is reduced due to damages from pollution x. Actual output (all of which is consumed) is $y = c = f\left(k_p\right)\left(1 - d\left(x; k_g\right)\right)$. The damage function d represents the fraction of potential output that is lost because of pollution x, and $d_x > 0, d_{xx} > 0$ (damages are increasing and convex in pollution). Geoengineering k_g affects damages also, with $d_k < 0$ and $d_{xk} < 0$. That is, geoengineering reduces absolute and marginal damages. Pollution x is determined by the total capital endowment and the fraction abated μ, so that $x = (1-\mu)k$. The fraction abated is a function of abatement capital: $\mu = g(k_a)$, where $g' > 0, g'' < 0$.

The planner's problem is to allocate the capital stock so as to maximize actual output (or equivalently maximize a monotone utility function over actual output). That is,

$$\max_{k_p, k_a, k_g} f(k_p)\left(1 - d(x; k_g)\right)$$

such that

$$k = k_p + k_a + k_g$$

$$x = \left(1 - g(k_a)\right)k$$

First, consider the constrained solution to this problem that omits GE, or sets $k_g = 0$. The solution to this constrained problem is analogous to policy recommendations by IAMs that ignore GE. The first-order condition for the constrained problem, assuming interior solutions, is

$$f'(k_p^c)\left(1 - d(x^c; 0)\right) = f(k_p^c)kg'(k_a^c)d_x(x^c; 0), \qquad (1)$$

where k_p^c, k_a^c, and x^c indicate solutions to the constrained problem. The left-hand side of equation (1) is the marginal cost of an additional unit of abatement, which is the foregone marginal output that could have been produced by allocating to production k_p instead of abatement k_a. The right-hand side is the marginal benefit of an additional unit of abatement, which is the reduction in damages caused by pollution from the extra unit of abatement. In a decentralized economy, the right-hand side of this equation is the optimal pollution tax, i.e. the social cost of carbon, when GE is ignored (as it is in many IAMs).

Next, consider the unconstrained problem where GE is not fixed at zero. This solution is characterized by two first-order conditions:

$$f'(k_p^{opt})\left(1 - d(x^{opt}; k_g^{opt})\right) = f(k_p^{opt})kg'(k_a^{opt})d_x(x^{opt}; k_g^{opt}) \qquad (2)$$

$$d_x(x^{opt};k_g^{opt})g'(k_a^{opt})k = -d_k(x^{opt};k_g^{opt}) \quad (3)$$

where $k_p^{opt}, k_a^{opt}, k_g^{opt}$, and x^{opt} indicate solutions to the unconstrained problem (i.e. the optimal levels). Equation (2), as in equation (1) in the constrained case, equates the marginal cost of an additional unit of abatement with its marginal benefit. In a decentralized economy, the optimal carbon tax (the social cost of carbon) is the right-hand side of equation (2). In equation (3), the left-hand side represents the marginal benefit of an additional unit of abatement (divided through by potential output f), and the right-hand side represents the marginal benefit of an additional unit of GE.

Consider the market for abatement k_a under both the constrained and unconstrained problem, as shown in the top half of Figure 1. The x-axis is the amount of abatement. The line $MC|k_g = 0$ is the marginal cost of abatement conditional on no GE ($k_g = 0$); it equals $f'(k - k_a)\left(1 - d\left(\left(1 - g(k_a)\right)k, 0\right)\right)$, which is the left-hand side of equation 1 evaluated at arbitrary k_a. The line $MB|k_g = 0$ is the marginal benefit of abatement with no GE; it equals $f(k - k_a)kg'(k_a)d_x\left(\left(1 - g(k_a)\right)k, 0\right)$. The appendix shows that MC is increasing and MB is decreasing in k_a. Where these are equal, their value is the social cost of carbon, conditional on no GE, as indicated by $SCC|k_g = 0$; this is the price determined by equation 1.

Allowing for GE affects the marginal benefit of abatement, and intuition suggests that the marginal benefit curve allowing for GE will be lower than the marginal benefit curve with no GE. GE reduces the damages from a unit of emissions, and therefore it reduces the marginal benefits of abatement. This intuition is verified in the appendix, and thus the curve $MB|k_g^{opt}$, which is the marginal benefit of abatement conditional on the optimal level of GE, is drawn in Figure 1 lower than $MB|k_g = 0$. Assuming that introducing GE does not change the marginal cost of abatement (an incorrect assumption, as it turns out, but one that we will maintain for now to clarify the point of Figure 1), then the social cost of carbon condition on optimal GE is lower than the social cost of carbon ignoring GE. Figure 1 demonstrates the deadweight loss in the abatement market (the triangle labeled DWL) from setting a carbon tax that ignores GE.

Allowing for GE also affects the marginal cost curve, though in Figure 1 we have ignored the fact that $MC\big|k_g = 0 \neq MC\big|k_g^{opt}$. The appendix shows that $MC\big|k_g = 0 < MC\big|k_g^{opt}$. This is because, for any abatement level, allowing GE makes the damages from pollution lower, and thus potential output higher, and therefore the marginal cost of abatement (foregone output) higher. But, the appendix also argues that the difference between the two marginal cost curves is likely to be small, unlike the difference between the two marginal benefit curves, which is why we have ignored the change in MC in Figure 1. Since the marginal cost under optimal GE is (slightly) higher than under no GE, the social cost of carbon under optimal GE will be closer to the social cost of carbon ignoring GE than shown in Figure 1 ignoring the change in marginal costs. But, the deadweight loss from ignoring GE could be higher or lower than that shown in Figure 1 (although the quantity of abatement will be lower, the cost of each unit over the optimal is higher).

The bottom half of Figure 1 shows that in equilibrium, the marginal benefit of an additional unit of abatement capital will equal the marginal benefit of an additional unit of geoengineering. The left-hand side is the market for abatement, while the right-hand side presents the market for geoengineering. The curves in blue represent the marginal costs and benefits of geoengineering, at a constant level of abatement (equal to k_a^c). As k_g increases and geoengineering in implemented, the optimal level of abatement k_a decreases, so emissions increase. Thus, the marginal benefit of each unit of geoengineering increases, since more pollution is allowed and temperatures are warmer without geoengineering. This is represented by an upward shift in the marginal benefit curve to the red curve. In equilibrium, the marginal benefits of geoengineering (on the right half) will increase just enough and the marginal benefits of abatement (on the left half) will decrease just enough so that the optimal quantity of each choice variable is such that the marginal benefits are equal across the two markets. Extending the new optimal carbon price across the graphs will intersect the equilibrium in the geoengineering market.

II.A. Uncertainty

We now amend the model to include uncertainty. Suppose that there are two random variables that affect the damage function *d*: call them θ_x and θ_g. Intuitively, θ_x represents uncertainty about the relationship between pollution and damages. For instance, this might represent uncertainty about the climate sensitivity parameter – how much temperature would increase after a doubling of atmospheric carbon. This uncertainty θ_x could also represent uncertainty over how temperatures affect the economy. The other shock, θ_g, represents uncertainty over geoengineering – either its implementation costs, its efficacy in controlling temperatures, or its negative side effects. Realistically, the third of these is the major source of uncertainty regarding GE.

Mathematically, we assume that the variance of either of these shocks affects marginal damages in the following way: $\frac{\partial E[d_x]}{\partial Var(\theta_x)} > 0$ and $\frac{\partial E[d_k]}{\partial Var(\theta_g)} > 0$. A higher variance in θ_x means that the expected marginal damages from pollution are higher. This could arise from the damage function itself, or it could reflect risk aversion in preferences, where the damage function d incorporates that risk aversion. A higher variance in θ_g increases d_k, that is, it makes it less negative – so it reduces the marginal benefits from geoengineering. Again, this could arise from the form of the damage function itself, or it could reflect risk aversion. We also assume that $\frac{\partial E[d]}{\partial Var(\theta_x)} > 0$ and $\frac{\partial E[d]}{\partial Var(\theta_g)} > 0$: both of these shocks affect the expected level of damages, not just the derivative. We make no assumptions over the "cross" effect of the shocks: $\frac{\partial E[d_x]}{\partial Var(\theta_g)}$ or $\frac{\partial E[d_k]}{\partial Var(\theta_x)}$.

Given these uncertainties, the planner chooses an allocation of production, abatement, and geoengineering to maximize expected net output. The implicit function theorem can be used on the two first-order conditions to present comparative static results on how uncertainty in either pollution damages or in geoengineering effectiveness affects optimal policy. The details of the derivation are presented in the appendix; here we present the results.

The appendix shows that

$$\frac{\partial k_a}{\partial Var(\theta_x)} = \frac{1}{Det}\left\{\left[f'(k_p)\left(g'(k_a)k\frac{\partial E[d_x]}{\partial k_g} + \frac{\partial E[d_k]}{\partial k_g}\right)\right]\frac{\partial E[d]}{\partial Var(\theta_x)} + A\frac{\partial E[d_x]}{\partial Var(\theta_x)} + B\frac{\partial E[d_k]}{\partial Var(\theta_x)}\right\}$$

Here Det is the determinant of the Jacobian matrix of the first-order conditions and is positive, and A and B are positive terms defined in the appendix. The second and third terms in the brackets are signed and easily interpretable. Since uncertainty over climate damages increases the expected marginal damages from pollution ($\frac{\partial E[d_x]}{\partial Var(\theta_x)} > 0$), it increases optimal abatement. The increase in the uncertainty of pollution damages makes abatement more attractive. If uncertainty over climate damages also increases d_k – i.e. reduces the marginal benefits from GE – then through this channel it also increases optimal abatement. The first term in the above expression, which is multiplied by $\frac{\partial E[d]}{\partial Var(\theta_x)}$, has ambiguous sign. Its first component, $g'(k_a)k\frac{\partial E[d_x]}{\partial k_g}$, is negative. The extent to which geoengineering reduces the marginal damages from pollution – d_{kx} – means that climate uncertainty's effect on total expected damages serves to reduce optimal abatement. This is because more geoengineering will be employed, and the more that GE reduces marginal damages from pollution, the less abatement is needed. The second component of the coefficient on $\frac{\partial E[d]}{\partial Var(\theta_x)}$, $\frac{\partial E[d_k]}{\partial k_g}$, is positive so long as $d_{kk} > 0$ – that is, the marginal benefits of geoengineering are decreasing. Since the uncertainty over climate increases expected damages, increased use of GE will be less effective and so more abatement will need to be used to compensate, hence this effect makes $\frac{\partial k_a}{\partial Var(\theta_x)}$ positive.

Also, the appendix shows that

$$\frac{\partial k_g}{\partial Var(\theta_x)} = \frac{1}{Det}\left\{\left[f'(k_p)\left(-g''(k_a)kE[d_x] - g'(k_a)k\frac{\partial E[d_x]}{\partial k_a} - \frac{\partial E[d_k]}{\partial k_a}\right)\right]\frac{\partial E[d]}{\partial Var(\theta_x)} - C\frac{\partial E[d_x]}{\partial Var(\theta_x)} - D\frac{\partial E[d_k]}{\partial Var(\theta_x)}\right\}$$

Here the terms C and D are both positive and defined in the appendix. As with the equation for $\frac{\partial k_a}{\partial Var(\theta_x)}$ above, here the second and third terms are unambiguous. Since uncertainty over climate increases marginal damages from pollution, this effect decreases optimal GE – instead more abatement is used instead of GE. If uncertainty over climate increases d_k (reduces marginal benefits from GE), then this reduces optimal GE because GE is less effective. The first term, multiplied by $\frac{\partial E[d]}{\partial Var(\theta_x)}$, has ambiguous sign. The first two components, $-g''(k_a)kE[d_x] - g'(k_a)k\frac{\partial E[d_x]}{\partial k_a}$, are positive. As expected damages are higher with more climate uncertainty, this will lead to more optimal GE, since marginal damages are positive ($E[d_x]$) and increasing ($\frac{\partial E[d_x]}{\partial k_a}$). The remaining component multiplying $\frac{\partial E[d]}{\partial Var(\theta_x)}$, $-\frac{\partial E[d_k]}{\partial k_a}$, is negative since $d_{kx} < 0$. Because increased use of GE will decrease marginal damages from pollution, there is an effect making optimal use of GE lower – less is needed since marginal damages are lower.

Comparing the two sets of ambiguous terms multiplying $\frac{\partial E[d]}{\partial Var(\theta_x)}$ in both of the above equations yields the following conclusion: if the cross-partial derivative d_{kx} is not too large in magnitude, then higher uncertainty in climate damages will increase use of both abatement and GE. That is, there is a scale effect since expected damages are larger, so it is optimal to use more of both tools available. The negative cross-partial derivative d_{xk} means that each of the two policy tools (abatement and GE) makes the other less effective, and thus this effect serves to reduce the use of each.

The equations for the effect of uncertainty in geoengineering on optimal policy are identical to the equations above, except with partial derivatives with respect to $Var(\theta_g)$:

$$\frac{\partial k_a}{\partial Var(\theta_g)} = \frac{1}{Det}\left\{\left[f'(k_p)\left(g'(k_a)k\frac{\partial E[d_x]}{\partial k_g} + \frac{\partial E[d_k]}{\partial k_g}\right)\right]\frac{\partial E[d]}{\partial Var(\theta_g)} + A\frac{\partial E[d_x]}{\partial Var(\theta_g)} + B\frac{\partial E[d_k]}{\partial Var(\theta_g)}\right\}$$

$$\frac{\partial k_g}{\partial Var(\theta_g)} = \frac{1}{Det}\left\{\left[f'(k_p)\left(-g''(k_a)kE[d_x] - g'(k_a)k\frac{\partial E[d_x]}{\partial k_a} - \frac{\partial E[d_k]}{\partial k_a}\right)\right]\frac{\partial E[d]}{\partial Var(\theta_g)} - C\frac{\partial E[d_x]}{\partial Var(\theta_g)} - D\frac{\partial E[d_k]}{\partial Var(\theta_g)}\right\}$$

As before, there are unambiguous effects from how uncertainty affects the first derivatives. Since $\frac{\partial E[d_k]}{\partial Var(\theta_g)} > 0$ (uncertainty about GE reduces expected marginal benefits of GE), more uncertainty about GE leads to less GE and more abatement; GE is a less-attractive option, and abatement is a substitute for it. Note that $Var(\theta_g)$ could also be taken to represent uncertainty about damages from GE; for example, the possibility that GE will damage the ozone layer. In fact, it is this aspect of d_k that motivates most of the uncertainty around GE technology. We show that the more uncertainty over this, the less GE will be used at the optimum.

Our results follow from the simplicity of our model, and it may be the case that in a model with more realistic features this result changes. For this reason we incorporate uncertainty over both climate damages and GE into our numerical simulation model below.

II.B. Decomposition of Climate Damages

We now amend the model to consider that damages from climate change may occur both from temperature changes, which GE addresses, and from carbon concentrations, which GE does not address. To model this simply, we separate the damage function into two components, only one of which is affected by geoengineering: $d(x; k_g) = \lambda_d d_1(x) + d_2(x; k_g)$. When $\lambda_d = 0$, this becomes the original model. But when $\lambda_d > 0$, there is a separate component of damages that cannot be alleviated with GE, and so each unit of GE is less effective at reducing damages from pollution. Damages that occur from temperature change, which GE can remedy, are modeled by d_2, and damages from carbon concentrations, which GE cannot remedy, by d_1. We conduct comparative statics on how this value affects optimal abatement and GE policy. When the fraction of climate damages from carbon, rather than from temperature, increases, λ_d will increase.

The appendix shows that

$$\frac{\partial k_a}{\partial \lambda_d} = C \cdot [g'(k_a)kd_{2xk} + d_{2kk}]$$

$$\frac{\partial k_g}{\partial \lambda_d} = C \cdot [(-g''(k_a)kd_{2x} + g'(k_a)^2k^2d_{2xx} + kg'(k_a)d_{2xk})]$$

The constant C is defined in the appendix and is positive.

In the equation for $\frac{\partial k_a}{\partial \lambda_d}$, the first term in brackets is negative, and the second term is positive. As more climate damages come from carbon rather than temperature (higher λ_d), the second (positive) term reflects the fact that more abatement is warranted, since it is the only approach that addresses carbon. In the equation for $\frac{\partial k_g}{\partial \lambda_d}$, the first two terms in brackets are positive, and the third term is negative. As more abatement is used because of a higher λ_d, there is less need for GE to alleviate d_2 because the cross-partial derivative is negative. This is captured in the third (negative) term. However, in each of the above equations there is a term or terms of opposite sign to the above intuition. The negative term in $\frac{\partial k_a}{\partial \lambda_d}$ reflects the fact that, as more abatement is employed to counter increased damages from carbon (d_1), the damages from temperature (d_2) are less intensive and therefore somewhat less abatement may be needed. The positive terms in $\frac{\partial k_g}{\partial \lambda_d}$ reflect the fact that an increase in λ_d increases the magnitude of climate change damages overall, and some of that can be alleviated with increased GE. This effect will be larger as d_{2x} and d_{2xx} are larger; that is, as marginal damages from temperature are greater and increasing.

The model in this section provides intuition but omits many important details. It is a static model, though climate change is inherently dynamic. It has no scientifically-based specification of how emissions affect the climate over time. For these reasons and many others, in the following section we incorporate GE into an IAM that includes these features. Though the output of the IAM is difficult to interpret intuitively (it is a "black box"), the intuition developed in this section will be carried over to the results from the IAM simulations.

III. Geoengineering and DICE

The dynamic integrated climate-economy (DICE) model by William Nordhaus is an IAM designed to be used to solve for optimal GHG abatement policy and calculate the social cost of carbon. It includes a representative-agent economic model with an endogenous capital stock and an exogenous level of technological growth in total factor productivity. Carbon emissions are a byproduct of production but can be reduced through expenditure on abatement. The climate component of the model includes several equations describing the dynamic interaction between carbon concentrations in several layers: the atmosphere and upper and lower oceans. The atmospheric carbon concentration affects the atmosphere's radiative forcing; that is, the difference between the amount of heat energy absorbed by the Earth and that radiated back into space. The human-caused change in radiative forcing is ultimately what affects atmospheric temperatures. Finally, the climate and economy sections of the model are integrated in that increases in temperature cause reductions in total economic output. A social welfare function is defined over consumption and output, and the model can be run to solve for optimal (i.e. welfare-maximizing) carbon abatement trajectories. Given marginal abatement costs, the social cost of carbon is a byproduct of the model's output. A time period in the 2007 version of the model is one decade, and the model is typically run over several dozen periods (hundreds of years).

The DICE model and its results have been refined over the years, and summaries of the model's equations and results are available in Nordhaus (2008) as well as Nordhaus's personal webpage.[9] A key takeaway from the model runs are that the social cost of carbon in the present is positive (typically around \$30 per ton of CO_2), and it is gradually increasing over time to reflect the increase in carbon concentrations and thus in marginal damages per ton of carbon.

IAMs like DICE have been criticized. Pindyck (2013) argues that they tell us "very little" and are "close to useless" because so many of the calibrated parameter values are ad hoc with little empirical foundation. This is demonstrated by the fact that the policy recommendations can be so sensitive to arbitrarily chosen parameter values, for instance the discount rate. Because our numerical analysis relies on DICE, it is subject to these criticisms. However, even if one accepts these critiques and is skeptical of IAMs, we argue that our analysis has merit. Though the point estimates of optimal policy paths should be interpreted with caution, how they vary with parameter values (i.e. the sensitivity analysis) still provides insight. Further, thought the point estimates may be problematic, the simulations demonstrate that it is potentially important to consider GE in optimal policy design, with or without IAMs. In these respects, the use of DICE can be seen as another argument in favor of Pindyck's (2013) and others' critiques of IAMs.

Here, we do not present all of the equations in the DICE model. Instead, we present the equations that we have modified to account for the possibility of geoengineering. We have modified DICE in the following five ways. First, while the only choice or action variable in the original DICE model is carbon abatement, we add a second choice variable to reflect the choice of the intensity of geoengineering. Second, there is a cost to implementing geoengineering, analogous to the cost of abatement. Third, we add potential damages from geoengineering, analogous to the original model's specification of damages from climate change. Fourth, the benefits of geoengineering are modeled as

[9] http://www.econ.yale.edu/~nordhaus/homepage/

directly modifying the radiative forcing equation. Fifth, we decompose the damages from climate change so that they depend directly not just on temperature, but also on atmospheric and on ocean carbon concentrations. In the following five paragraphs we describe in detail each of these modifications.

The original DICE model's sole choice variable is abatement intensity a, which can take values between zero and one. When $a = 0$, there is no abatement, and $a = 1$ implies that all carbon emissions are abated (no emissions). Net carbon emissions are $E_{net} = (1 - a)E_{gross}$. We add a second choice variable for the intensity of geoengineering, g (and we maintain the choice of abatement). When this variable g equals zero, this represents no geoengineering. When $g = 1$, this represents "full" geoengineering, i.e. fully offsetting the warming effects from increased carbon concentrations (described in more detail below). However, unlike abatement intensity a, geoengineering intensity g could take a value larger than 1, representing more than fully offsetting temperature increases from climate change.

In the original DICE model, the cost of abatement is modeled as a power function of a: $AbatementCost = \theta_1(t)a^{\theta_2}$. The exponent θ_2 is 2.8 in the base case, indicating convex costs. The coefficient $\theta_1(t)$ decreases with time, halving after about 100 years (10 periods) to reflect technological advancement in abatement. The outcome variable $AbatementCost$ is the fraction of gross output that is sacrificed for abatement. For instance, in period 1 where $\theta_1(1) = 0.0561$, the cost of abating 10 percent of gross emissions would be 0.009% of gross output ($0.00009 = 0.0561 \times 0.10^{2.8}$). We define geoengineering costs analogously as a fraction of gross output: $GeoengCost = G_{coef}\theta_{GE}(t)g^{\theta_3}$. To calibrate this cost function of aerosol-sulfate-based climate engineering, we use two sources.[10] First,

[10] There are alternatives solar radiation management technologies other than sulfate aerosols, though sulfate aerosols are likely the most cost-effective and dependable technology. Marine cloud brightening (MCB) would increase reflectivity by injecting seawater particles into clouds (Latham et al. 2014b). Cirrus cloud seeding would increase outgoing radiation by reducing cirrus cloud cover (Storelvmo et al. 2014).

doing well-informed back-of-the-envelope calculations, Crutzen (2006) estimates the amount of sulfur needed to reduce the radiative forcing equivalent to doubling CO_2 to be equal to 5.3 Mt of sulfur. That is, we can reduce a radiative forcing equivalent to 4W/m^2 by deploying 5.3 Mt S. The second piece of information is related to the costs of delivering sulfur at the distance required to have a global impact. Crutzen (2006) has estimated something in the order of $25 Billion for 1 Mt S. Recent estimates, using new aircraft designs, estimated the costs at $3 Billion for 1 Mt S or $8 Billion to deliver 5Mt S (McClellan et al. 2012). These two pieces of data imply that reducing the radiative forcing equivalent to a doubling of CO_2 costs between $8 Billion and $125 Billion. Furthermore, we assume that particles are required at an increasing rate (Rasch et al. 2008), and for simplicity the costs are quadratic (less convex than mitigation, but linear costs are unrealistic due to coagulation of particles and other such processes). GDP in 2005 was $46 Trillion, so the lowest estimate of $8 Billion is only 0.02% of global GDP and the highest estimate is 0.27% of global GDP. (Compare this with the 3% in terms of mitigation costs associated with the optimal policy in DICE.) Because geoengineering is a fraction in our model, then reducing a doubling of CO_2 to nothing is equivalent to setting $g = 1$. Thus our geoengineering cost estimate is $\theta_{GE}(t) = 0.0027$ and $\theta_3 = 2$. We set $\theta_{GE}(t)$ constant over time, so that unlike with abatement technology there is no learning or improvement in geoengineering technology. We also include the coefficient G_{coef} to represent a scaling of geoengineering costs. In the base case we set $G_{coef} = 1$, and we will vary this in sensitivity analysis. By using the high cost estimate and not allowing technological growth in GE technology, this base case value for GE costs is very conservative, that is, biased against deployment of geoengineering. Keith et al. (2010) argues that "long-established estimates" show that solar radiation management GE can offset climate change at least 100 times more cheaply than the cost of abatement.

In addition to these implementation costs of geoengineering, there may also be damages from geoengineering. For instance, sulfates are expected to exacerbate ozone depletion (Heckendorn et al. 2009). Precipitation could be drastically reduced (Ferraro et al. 2014, Robock et al. 2008). The sulfates injected into the stratosphere may condense and fall back to the atmosphere, contributing to acid rain (though Kravitz et al. (2009) find that this effect will be insubstantial). We model these damages in the same way that the original DICE model models damages from climate change – as a factor of total potential output that is unrealized due to these damages (and of course we keep damages from climate change as well). In the original DICE, the damage function is $\Omega(t)$, where output $Q_{net} = \frac{1}{1+\Omega(t)} Q_{gross}$ and $\Omega(t) = \psi_1 T_{AT}(t) + \psi_2 [T_{AT}(t)]^2$. The damage function is a function of atmospheric temperature at time t, $T_{AT}(t)$, and the ψs are calibrated coefficients. We amend this by also allowing for geoengineering g to directly reduce total output. In addition to the $\Omega(t)$ term representing damages from climate change, we also include damages from geoengineering: $Q_{net} = \frac{1}{1+\Omega(t)} \times \frac{1}{1+\nu_G g^2} \times Q_{gross}$. The coefficient ν_g represents how damages from geoengineering scale net output. In our base case, we set $\nu_g = 0.03$, which implies that geoengineering at full intensity ($g = 1$) leads to damages that amount to 3% of gross output. This is similar in scale to the damages from climate change damages in DICE associated with about 6 degrees Celsius of warming, thus this damage estimate (like the cost estimate also) is very conservative (i.e. biased *against* geoengineering).

The purpose of engaging in solar geoengineering is to alter the radiative forcing of the Earth's atmosphere. Radiative forcing is the difference in net heat loss due to anthropogenic GHG emissions relative to preindustrial levels. In the DICE model, the radiative forcing equation is

$$F(t) = \eta\{\log_2 \left[\frac{M_{AT}(t)}{M_{AT}(1750)}\right]\} + F_{EX}(t)$$

It is a function of the ratio of the current atmospheric carbon stock ($M_{AT}(t)$) to the pre-industrial atmospheric carbon stock ($M_{AT}(1750)$ = 596.4 Gt C, equivalent to about 280 ppm CO_2), exogenous forcing $F_{EX}(t)$ due to anthropogenic emissions of GHGs other than CO_2 (assumed exogenous in DICE), and a calibrated radiative forcing parameter η. Then, atmospheric temperature $T_{AT}(t)$ is affected by radiative forcing through the following equation:

$$T_{AT}(t) = T_{AT}(t-1) + \xi_1\{F(t) - \xi_2 T_{AT}(t-1) - \xi_3[T_{AT}(t-1) - T_{LO}(t-1)]\}.$$

This function also depends on the lower ocean temperature in the previous period $T_{LO}(t-1)$. A higher value of radiative forcing $F(t)$ (which is caused by higher atmospheric carbon $M_{AT}(t)$) leads to higher atmospheric temperatures $T_{AT}(t)$ all else equal. Our modification is to the radiative forcing equation:

$$F(t) = \left(\eta\{\log_2\left[\frac{M_{AT}(t)}{M_{AT}(1750)}\right]\} + F_{EX}(t)\right)(1 - \phi g(t))$$

The variable $g(t)$ is the extent of geoengineering in period t, and ϕ is a positive parameter that captures the leverage of geoengineering. Higher ϕ means less geoengineering needs to be implemented to achieve a given level of radiative forcing reduction. At a value of 1, radiative forcing F is reduced to zero (regardless of the carbon stock), completely eliminating anthropogenic climate change effects on temperature.

Our final change to the DICE model is a modification of the damage function from climate change (not from geoengineering). In DICE, climate change damages are a function of temperature only: $\Omega(t)$. However, climate change damages are expected to come from more than just temperature changes. precipitation is predicted to change, independent of the temperature change due to global warming. Bony et al. (2013) find that atmospheric carbon increases account for about half of predicted tropical circulation change and a large fraction of precipitation changes from climate change. The acidification of oceans cause damages too. There is also the possibility that increased atmospheric carbon concentrations may have benefits to agricultural productivity, holding temperature and precipitation constant (Pongratz et al. 2012). Because geoengineering decreases temperatures but does not change the carbon stock in either the atmosphere or the oceans, it is crucial to decompose the damages from climate change into damages directly from temperature and damages from carbon stocks. Thus, we modify DICE's damage function $\Omega(t)$ to be a function of temperature $T_{AT}(t)$ as well as atmospheric carbon $M_{AT}(t)$ and upper ocean carbon $M_{UP}(t)$:

$$\Omega(t) = \psi_T[T_{AT}(t)]^2 + \psi_O[M_{UP}(t) - M_{UP}(1750)]^2 + \psi_{AT}[M_{AT}(t) - M_{AT}(1750)]^2$$

The temperature T_{AT} is already defined as degrees Celsius relative to preindustrial (1750) average temperature, but the other two components of damages are not, so we calculate damages by subtracting the preindustrial levels, and we allow damages to be a quadratic function of the deviation from preindustrial levels. We calibrate the new damage coefficients ψ in the following way. We impose that the original DICE model's total climate change damages in the initial period is correct, but we allocate some of those damages to be directly from temperature, some from ocean carbon concentrations, and some from atmospheric carbon concentrations. Brander et al. (2012) suggest that find that damages from ocean acidification alone can account for 0.14% to 0.18% of gross GDP. Direct damages from atmospheric carbon concentrations are difficult to calibrate. Absent its effect on temperature, atmospheric carbon is expected to affect precipitation patterns, which may cause damages.(Allen 2002, Bala et al 2008) Furthermore, atmospheric carbon may provide a benefit to agriculture by promoting more robust crop growth. (Matthews et al. 2005) Therefore, we begin by assuming that, of the total damages from climate change in the initial period calibrated in DICE, 80% is directly a function of atmospheric temperature, 10% is a function of atmospheric carbon, and 10% is a function of upper ocean carbon. By imposing that the total damages from climate in the initial period are identical to those in DICE, this allows us to calibrate ψ_T, ψ_O, and ψ_{AT}. Specifically, in the initial period in DICE, climate change damages amount to 0.15157% of gross output ($\Omega(t) = \psi T^2 = 2.8388 \times 10^{-3} \times 0.7307^2$). We set 80% of this total a function of temperature, yielding $\psi_T = 0.00227$ ($\psi_T \times 0.7307^2 = 0.80 \times 0.0015157$). And similarly we can calibrate the other damage parameters.

These are the five areas in which we modify DICE to include geoengineering and its costs, benefits, and damages. We have described the base case parameterizations that we use in our model, but we will also conduct a very broad range of sensitivity analyses, since many of the parameters are difficult to quantify.

Other studies have also modified DICE to include geoengineering, and here we compare our modifications to these other papers'. Table 1 summarizes this comparison. All of the papers allow solar GE to directly modify the radiative forcing equation; our paper is the only one to allow GE to enter as a multiplicative factor rather than a linear additive term. This is not a major difference, since in either case the substantive effect is to reduce the value of $F(t)$. We choose a multiplicative factor for ease of interpretation: the policy variable will be between zero and one, representing the fraction of total anthropogenic forcing that is eliminated via solar GE. The next column notes that all of the previous studies except one allow for damages from solar GE (apart from their implementation costs). We, like Gramstad and Tjotta (2010), allow for these damages to be a quadratic function of GE intensity and to be a multiplier on gross output; in this way they are modeled analogously to damages from climate change. Next, only this paper and Goes et al. (2011) and Bickel and Agrawal (2013) modify DICE's damages from climate change function. The other two papers allow for damages to be a function both of temperature and of the rate of temperature change, based on the fact that solar GE can lead to rapid temperature changes (Matthews and Caldeira 2007). Their damage function is taken from Lempert et al. (2000). We are more direct in that we allow for damages to be a function of more than just temperature; this innovation is unique to this paper.

There are other modifications as well. Bickel and Lane (2009) is the only paper that also considers carbon capture GE, and Goes et al. (2011) and Bickel and Agrawal (2013) make several other modifications, including using a different climate model altogether.

A unique contribution of this paper is to treat the underlying uncertainty in the climate system by adopting a stochastic optimization, rather than conventional sensitivity analyses or Monte Carlo (MC) simulation. The advantage of this approach is that unlike sensitivity analysis, it incorporates the prior knowledge about probability distributions of uncertain parameters into the solution method. Moreover, although it features a similar random sampling as in MC simulation, the numerical results are used to develop the optimal strategy rather than demonstrating the range of possible outcome. The optimal strategy then can be used to produce the prediction for any new realization of the uncertain parameter. This is the key advantage of stochastic optimization techniques over conventional MC simulation: in stochastic optimization (or Reinforcement Learning in computer science language) the agent can update its optimal decision in face of uncertainty based on a large (finite) number of observations of random realizations of uncertain parameter and outcome but in MC simulation. The other GE DICE papers consider sensitivity analyses, but they do not model stochastic parameters. There are several papers including Baker and Solak (2011) and Kolstad (1996) that had modified DICE to include stochastic parameters, but without GE. There are several other studies that have used different approximation techniques to address the uncertainty in the DICE model framework. Kelly and Kolstad (1999) used neural network approximations for obtaining flexible functional form of the value function. Oppenheimer et al. (2008) discretized the uncertainty over climate sensitivity and solved the stochastic DICE model in discrete deterministic stages. Webster et al. (2012) used two parametric and non-parametric methods to approximate the value function. Cai et al. (2014) applied Chebyshev polynomial approximation for value function estimation. In this paper we consider continuous state and probability spaces in our model and adopt a unique stochastic approximation technique for finding the optimal strategy in the face of uncertainty in the climate system and GE deployment. We do not “invent” a new approximation function and we use the already calculated functions within the model as building blocks of our value function approximation. This reduces the number of tunable parameters and substantially limits the subsequent optimal search domain. Furthermore, this algorithm is intuitive in the sense that it forecast a limited number of steps ahead given the current realization of the uncertain parameter and use this as an insight to make a prediction about the values of future states.

The model is solved for optimal policy using the two-step-ahead approximation method described in Shayegh and Thomas (2015). This algorithm was originally developed to find the optimal solution for the stochastic case of uncertainty in the climate sensitivity parameter in DICE. The approximation technique was tested and tuned in the deterministic case and then applied to the stochastic model. The algorithm works as follows: at each time step t, the decision maker is projecting the future values of uncertain variables for the next two steps. At any specific state the value function is approximated by projecting the values of states in the next two time periods. The values of the two projected states are calculated under a deterministic forecast and brought back to the present time using an artificial and tunable discount rate. These values reflect the social utility under the deterministic assumption and are used to construct the value function of the current state. The optimal action (abatement, geoengineering, etc.) is found by maximizing this value function. The algorithm starts at time $t = 1$ and progress until the last time step. In order for the model to update the decision rule for taking optimal actions, we need to find the best set of parameters for the value function approximation. At each iteration, assuming a constant set of parameters, the value functions are used to approximate future values and derive optimal actions. Once an iteration is complete, we can move backward and calculate the "actual" values of future states given such actions. The difference between these actual and estimated values (generated from value function approximation) constitutes an error margin for that iteration. For the next iteration we update the value function approximation parameters to close the gap between actual and estimated values. The algorithm iterates until it "learns" the decision rule (i.e. finds the best value function approximation). The iteration ends when the error (difference between old and new values) converges to zero.

Although it is designed for dealing with uncertainty in stochastic problems, the algorithm can be used in a deterministic model to approximate the optimal solution. We adopt this algorithm and modify it for the case of two actions: abatement and geoengineering. The algorithm is developed in Matlab and is available upon request.

IV. Simulation Results

IV.A Baseline Simulations

In order to understand the way geoengineering affects optimal climate policy, we start by analyzing the deterministic case. We compare the outcomes of the baseline scenario to the case of no geoengineering; that is, a model that does not allow for geoengineering. The results are presented in Figure 2. The first panel shows how abatement is affected when geoengineering is introduced as a viable policy instrument. The introduction of geoengineering lowers the level of abatement and delays the time when we transition to a clean economy. Once abatement reaches it maximum level, geoengineering begins to decline. However, it stays positive for some time because of the lag in the effect of emissions on temperature. The optimal GE deployment is a "ramping-up" policy, starting out at low levels and gradually increasing as the damages from climate change increase. Although GE is allowed to take a value greater than 1, its maximum value is just about one-half (i.e. offsetting half of the increase in radiative forcing from carbon concentrations). This is because the benefits from GE are traded off against the (substantial) damages. Eventually, GE use declines towards zero, since carbon concentrations are reduced. GE is a substitute for abatement in the short- and medium-run, but eventually abatement dominates.

The introduction of geoengineering, therefore, has important implications for climate outcomes. In the next two panels we look at carbon dioxide concentrations and temperature changes. Because of the lower level of abatement, carbon dioxide concentrations peak at a higher level and later in the presence of geoengineering. Concentrations peak at 1600ppm, relative to the case of no geoengineering where concentrations peak at 1400ppm. But with geoengineering, temperature peaks much earlier and it is kept at check below the 2 degrees mark. This is the buying-time effect, often cited in the literature, where geoengineering keeps the system below deleterious levels of climate change while the abatement technology improves enough to eliminate emissions (Keith 2014 and Moreno-Cruz and

Smulders, 2007). This is done at the cost of allowing for higher levels of concentrations. Thus, there is a tradeoff between carbon damages and temperature damages, as well as geoengineering costs and abatement costs.

The fourth panel in Figure 2 shows that the carbon price is lower when geoengineering is introduced; it peaks at a lower level before it starts to decline at the rate of learning by which the costs of the backstop technology decline. As the analytical model shows, ignoring geoengineering leads to a carbon price that is too high. After 100 years, the optimal carbon price is about 30% lower than the price from the model ignoring geoengineering; after 200 years it is about 45% lower.

What is remarkable about all these results is that they do not arise because geoengineering is very cheap, since we are very conservative about the costs and damages of GE. These results are due to the use of geoengineering directly on the radiative forcing. This reduces the inertia of the climate system, reducing the amount of abatement needed today to reduce concentrations in the future. Thus, by postponing costly abatement to future periods, geoengineering helps to reduce the aggregate costs of climate change. This is demonstrated in the last panel of Figure 2, which plots the costs in proportional GDP loss of ignoring geoengineering. For instance, at year 200, this value is 1.52%, indicating that net GDP (after accounting for climate damages, geoengineering damages, abatement costs, and geoengineering costs) is 1.52% lower in the "no geoengineering" simulation than it is in the baseline simulation. This corresponds to the area of deadweight loss from the analytical model in Figure 1.

These deterministic simulation results verify what we find in the analytical model – allowing for geoengineering reduces the optimal level of abatement, reduces the optimal carbon price, and reduces total policy costs.

IV.B Variation in the Composition of Damages – Temperature vs. Carbon

Because GE reduces temperatures without reducing atmospheric or ocean carbon concentrations, it cannot completely offset all damages from climate change. In the baseline specification, we assume that 80% of climate damages are directly from temperature, 10% are from atmospheric carbon concentrations, and 10% are from ocean carbon concentrations. In Figure 3, we present simulation results where we vary this decomposition of climate damages. In addition to the baseline case, we simulate three other damage decompositions, in each of which damages from temperature only account for 50% of climate damages. The remaining 50% of damages are split between atmospheric and ocean carbon 25/25, 40/10, or 10/40.

Comparing the baseline case to any of the three alternate decompositions shows that there is more geoengineering and less abatement when temperature accounts for a higher fraction of climate damages. GE is less effective relative to abatement when temperature accounts for less damages, and so less of it is deployed. This corresponds to the results from the analytical model that $\frac{\partial k_a}{\partial \lambda_d} < 0$ and $\frac{\partial k_g}{\partial \lambda_d} > 0$. Comparing the three alternative decompositions to each other shows that there is more geoengineering and less abatement when ocean carbon concentrations account for a higher fraction of damages than do atmospheric carbon concentrations. Abatement more directly affects atmospheric rather than ocean carbon, since the absorption of emitted carbon by the ocean is gradual and slow. If atmospheric carbon is more damaging than ocean carbon, more abatement and less GE is needed.

The actual composition of damages between ocean carbon, atmospheric carbon, and temperature is unknown. In fact, atmospheric carbon may yield benefits from increased agricultural productivity. The purpose of this analysis is not to provide policy recommendations but rather to demonstrate the importance of research on measuring these distinct damages from climate change and incorporating them into assessment models. For mitigation policy, the distinction is unimportant. But because GE severs the link between carbon and temperature, the distinction matters.

IV.C Uncertainty

We now allow DICE to be solved assuming a stochastic distribution of certain parameters. We allow the model to converge, and then solve the model 1000 times for different draws of the stochastic parameter based on a specified distribution. We then evaluate the mean of the policy outcome variables as well as their distribution. We can examine how, for instance, the uncertainty in parameter values affects both the mean of policy variables and the distribution of values. We also compare the solutions under uncertainty with the solutions in the deterministic case.

First, we allow the climate sensitivity parameter to be stochastic. This parameter describes the equilibrium temperature change that results from a doubling of atmospheric carbon. In our deterministic case this is set to 3. We now allow it to take on a truncated log-normal distribution, calibrated based on the IPCC report (IPCC 2013). The lower and upper bounds are 0.1 and 20, respectively; the mean and standard deviation are 1.1 and 0.55, respectively. This parameterization is used in Shayegh and Thomas (forthcoming).

Figure 4 presents the policy simulation results under uncertainty over climate sensitivity. For each of the policy outcomes (abatement, geoengineering, temperature, atmospheric carbon, and the price of carbon), we present the mean value (in red) across the 1000 simulations, the 5th and 95th percentiles, and the value from the deterministic case (in green). The green and red curves are very close to each other, indicating that the average policy outcome is not much different than the deterministic case (this is because the distribution of the stochastic climate sensitivity variable is set so that its average is the deterministic value). The 5th and 95th percentile values demonstrate that uncertainty over climate sensitivity affects optimal geoengineering policy much more so than it affects optimal abatement policy. The 5th to 95th percentile bands for abatement, carbon concentrations, and the carbon price are very narrow. This is surprising, since the only stochastic variable in this simulation is the climate sensitivity variable, which is not directly related to geoengineering.

We can also repeat these stochastic simulations under different distributions of climate sensitivity. In particular, we consider alternate values for the standard deviation of the parameter, both higher (1.0) and lower (0.20) than the base case (0.55). We find that the level of uncertainty of climate sensitivity does not substantially affect the mean values for abatement action or the carbon price, although it does (unsurprisingly) affect the percentile bands. A smaller standard deviation of beliefs about climate sensitivity leads to a narrower confidence interval. However, the standard deviation of climate sensitivity does affect both the mean and the percentile bands for optimal GE action. A larger standard deviation in climate sensitivity leads to a lower mean level of geoengineering action.

Next, we allow a different parameter to be stochastic. Figure 5 shows results from simulations in which the parameter ν_G, the coefficient on damages from GE, is stochastic. The potential damages from solar geoengineering represent the primary source of uncertainty.[11] The distribution of this parameter is assumed to be lognormal, with a median value of 0.03, identical to the value on the deterministic case. As in the case of uncertainty over climate sensitivity, here with uncertainty over GE damages, we find that uncertainty affects the distribution of optimal GE policy by a much greater amount than it affects the distribution of optimal abatement policy. Optimal GE can peak at anywhere between 20% and 110% intensity. As a result, temperatures can peak between 1 and 2 degrees above preindustrial levels.

IV.D Sensitivity analysis

Lastly we consider how variation in certain parameters affects optimal policy. In these simulations, presented in Figure 6, we conduct deterministic simulations for several different values of certain parameters, along with the no-geoengineering scenario. Figure 6 presents the optimal GE deployment path under each parameter value; the Appendix figures present the other policy outcomes

[11] National Research Council (2015).

(including abatement and the social cost of carbon). We vary the costs of geoengineering (G_{coef}, Panel A), its effectiveness at counterbalancing radiative forcing (ϕ, Panel B), the damages associated with its implementation (ν_g, Panel C), and the social discount rate (ρ, Panel D).

As the implementation costs of geoengineering increase, less geoengineering is deployed. Because these costs are so low in the base case, an order-of-magnitude change in the coefficient in front of these costs has only a modest effect on GE deployment; the maximum level of GE intensity varies from 20% to 50%. Appendix Figure 1 shows that abatement, carbon concentrations, and the social cost of carbon are not very sensitive to this parameter. But even if we make geoengineering 10 times more costly than the base case, there is a substantial amount of warming that is still compensated by geoengineering. This reflects the fact that by eliminating the inertia of the carbon cycle and therefore allowing for postponing abatement, geoengineering decreases the total costs of climate change and increases welfare.

Panel B of Figure 6 shows that geoengineering effectiveness affects its deployment – as GE is more effective, it is used more intensively. When its effectiveness is very low ($\phi = 0.01$), it is barely used. When it is very effective ($\phi = 5.0$), it is immediately ramped up to 25% intensity, after which it gradually increases. Even after 500 years we see no decline in its intensity. Appendix Figure 2 shows that more effective GE results in lower abatement and higher carbon concentrations, but lower temperatures. With a high effectiveness of $\phi = 2$, temperatures are brought back to pre-industrial levels after just 200 years.

Next, variation in the damages from GE cause a very wide range of optimal GE deployment, as seen in Panel C of Figure 6. When damages are an order-of-magnitude lower than the base case ($\nu_G = 0.003$), GE eventually reaches greater than 100% intensity. The variation from damages, in Panel C, is so much larger than the variation from costs, in Panel A, because costs are so small and damages (at least in our conservative calibration) are quite large. Appendix Figure 3 demonstrates that the variation in optimal abatement and carbon concentrations is smaller than the variation in GE deployment. With the lowest level of GE damages, mean temperatures are brought back to within 0.5 degrees of preindustrial levels by 200 years.

Finally, increasing the discount rate (Panel D of **Error! Reference source not found.** and Appendix Figure 4) decreases the amounts of both abatement and geoengineering, as well as the carbon price. Abatement and geoengineering are postponed to a later stage, but also less geoengineering is implemented overall and at its peak. This suggests that more patient societies would tend to favor abatement over geoengineering.

In all of these sensitivity analyses, the level of abatement varies but eventually reaches 100%. Can we find reasonable parameter values where abatement is never used in the long run? That is, geoengineering in theory could be so cheap that the optimal solution ends up at a corner solution with no use of abatement at all. However, even when we reduce the geoengineering costs to 1% of its base case value and geoengineering damages to 10% of its base case value, and we reduce the fraction of climate change damages attributable to atmospheric and ocean carbon to just 1% each, we are still left with a case where abatement eventually reaches 100% intensity. It is much delayed, not occurring until period 35 (relative to period 200 in the no-GE case). For as long as there are damages associated with carbon concentrations that are not related to temperature, there will be a need for abatement.

V. Conclusion

Solar geoengineering has the potential to lower the costs of dealing with climate change and reduce the need for high levels of abatement and a high carbon price. Three points are crucial. First, models that ignore geoengineering may prescribe policies that abate too much, cost too much, and have a carbon price that is too high. Second, uncertainty over both climate damages and geoengineering costs and damages can substantially affect optimal policy. Third, because solar geoengineering reduces temperatures but not carbon concentrations, it is merely an imperfect substitute for abatement. We explore these issue through both an analytical theoretical model and a numerical integrated assessment model of climate change. Our modification of the DICE model provides quantitative insights as to how geoengineering can affect optimal abatement policy. The level of abatement can be about 25% lower when allowing for geoengineering, and the optimal atmospheric carbon concentrations can be more than 20% higher. Despite that, temperature changes can be kept about one-and-a-half degrees Celsius lower because of the use of geoengineering, and total GDP losses can be lower by up to one-and-a-half percentage points of GDP. These base-case results are of course sensitive to the parameter values, which are very uncertain. Still, under a wide (two orders of magnitude) range in parameters describing the costs and damages of geoengineering, the optimal carbon price and level of abatement do not vary substantially, although the optimal level of geoengineering does vary substantially (ranging from nearly no geoengineering to more than 100% geoengineering). As with all climate models, more precise parameter values are essential for pinning down specific policy recommendations.

We caution that these results should not be interpreted as a policy prescription for immediate deployment of geoengineering. The uncertainties surrounding the calibration of the model, in particular the damages associated with geoengineering, are too great to be able to do so. Instead, the main contribution of this paper is in its qualitative and quantitative exploration of how including GE in climate models affects the optimal deployment of abatement and the social cost of carbon, of how uncertainty affects optimal policy, and of how important it is that geoengineering reduces temperatures but not carbon.

Still, the fundamental contribution made by this study has important policy implications. It is not efficient to merely estimate the marginal external damages of a ton of carbon and institute that carbon tax, if the external damages are estimated in a model without the possibility of geoengineering. Our results suggest that this may in fact be the case, and that for this reason the social cost of carbon currently being used by policymakers may be too high.[12] Of course, there are many other potential reasons why the social cost of carbon currently used may be too low – estimates may omit many benefits from carbon reductions.[13]

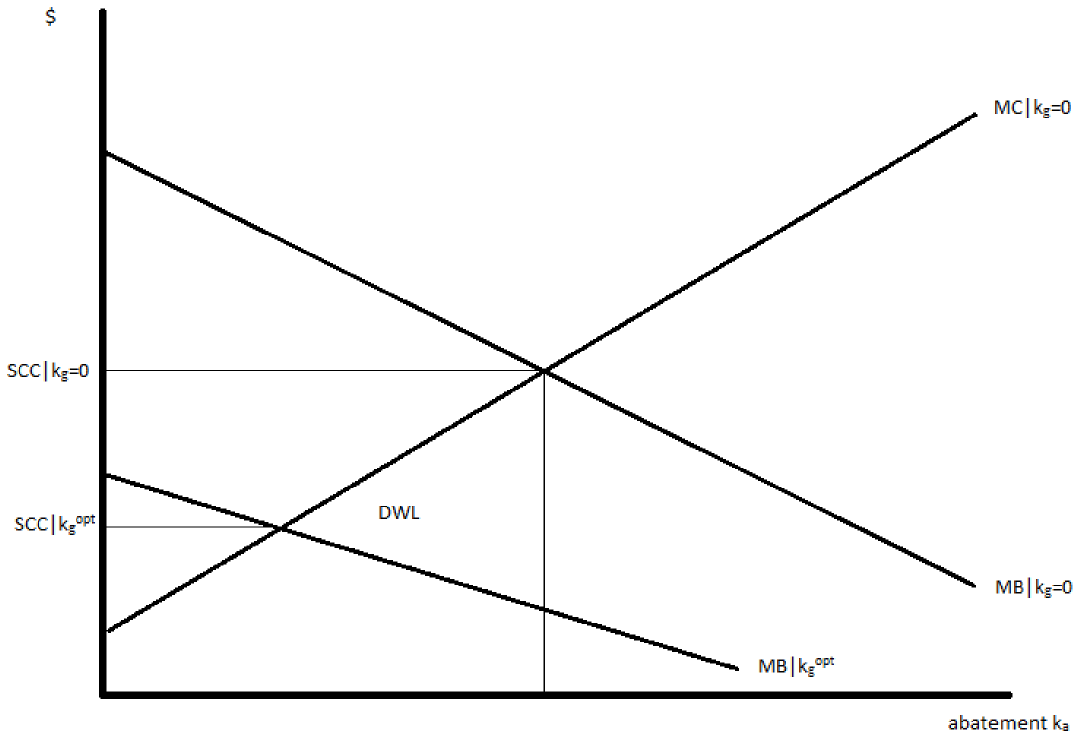
Our research emphasizes the need for more information on costs and benefits of geoengineering. Furthermore, refinements to the model may yield valuable policy lessons – for instance expanding the set of parameters modeled as stochastic variables, or adding refinements to either the climate model in DICE or its treatment of economic costs or growth. Future research in progress is examining how GE can address the issue of tipping points, or irreversibilities and discontinues in climate damages. The damages from GE represent probably the most "unknown" of all of the features of this model. Finally, there are many issues related to GE that we do not or cannot address using an IAM – including a fat-tailed distribution of risks, discontinues in costs of damages, and ethical issues related to the question of abatement versus GE.

[12] The EPA and other federal agencies use an SCC of $37 per ton of CO_2: http://www.epa.gov/climatechange/EPAactivities/economics/scc.html.

[13] See for instance Howard (2014).

Table 1 – Summary of modifications to DICE

	Radiative Forcing	Damages from Solar GE	Climate Change Damages	Other modifications	Outcomes
Bickel and Lane (2009)	Linear term in forcing equation	None	No Modifications	Also model carbon capture geoengineering	Cost-benefit analysis for fixed levels of GE; carbon price
Gramstad and Tjotta (2010)	Linear term in forcing equation	Quadratic multiplier on gross output	No Modifications	None	Cost-benefit analysis for fixed levels of GE
Goes et al. (2011) and Bickel and Agrawal (2013)	Linear term in forcing equation	Linear function of aerosols deployed	Damages a function of temperature and rate of temperature change	Alter discounting formula; change climate model to DOECLIM; intermittency in GE	Cost-benefit analysis for fixed levels of GE and for optimal GE/abatement mix; Bickel and Agrawal (2013) considers sensitivity analysis of Goes et al. (2011)
This paper	Multiplicative factor in forcing equation	Quadratic multiplier on gross output	Damages a function of temperature, atmospheric carbon, and ocean carbon	Stochastic analysis of climate sensitivity	Optimal levels of GE and abatement; carbon price; sensitivity analyses



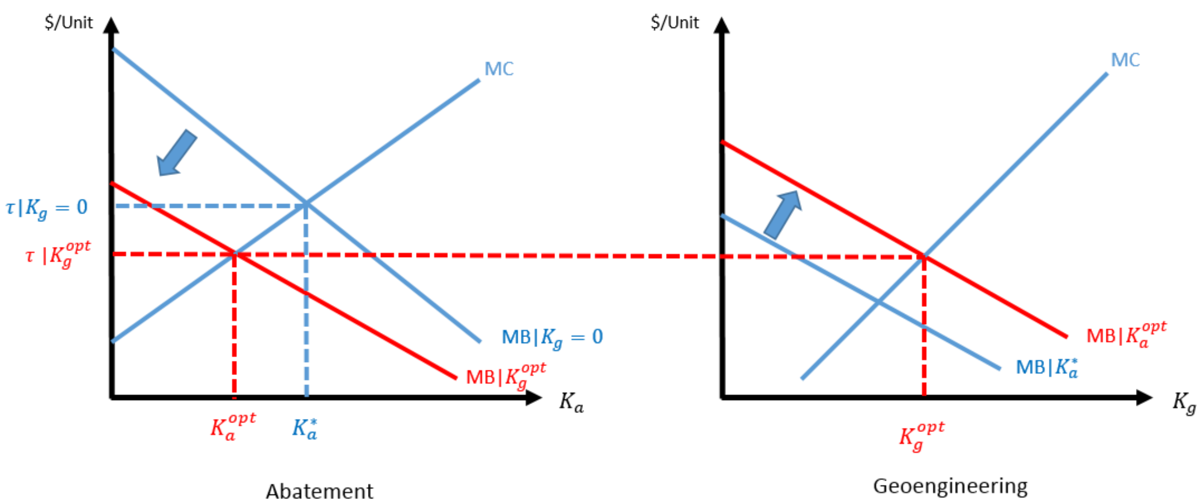


Figure 1 – Static Model

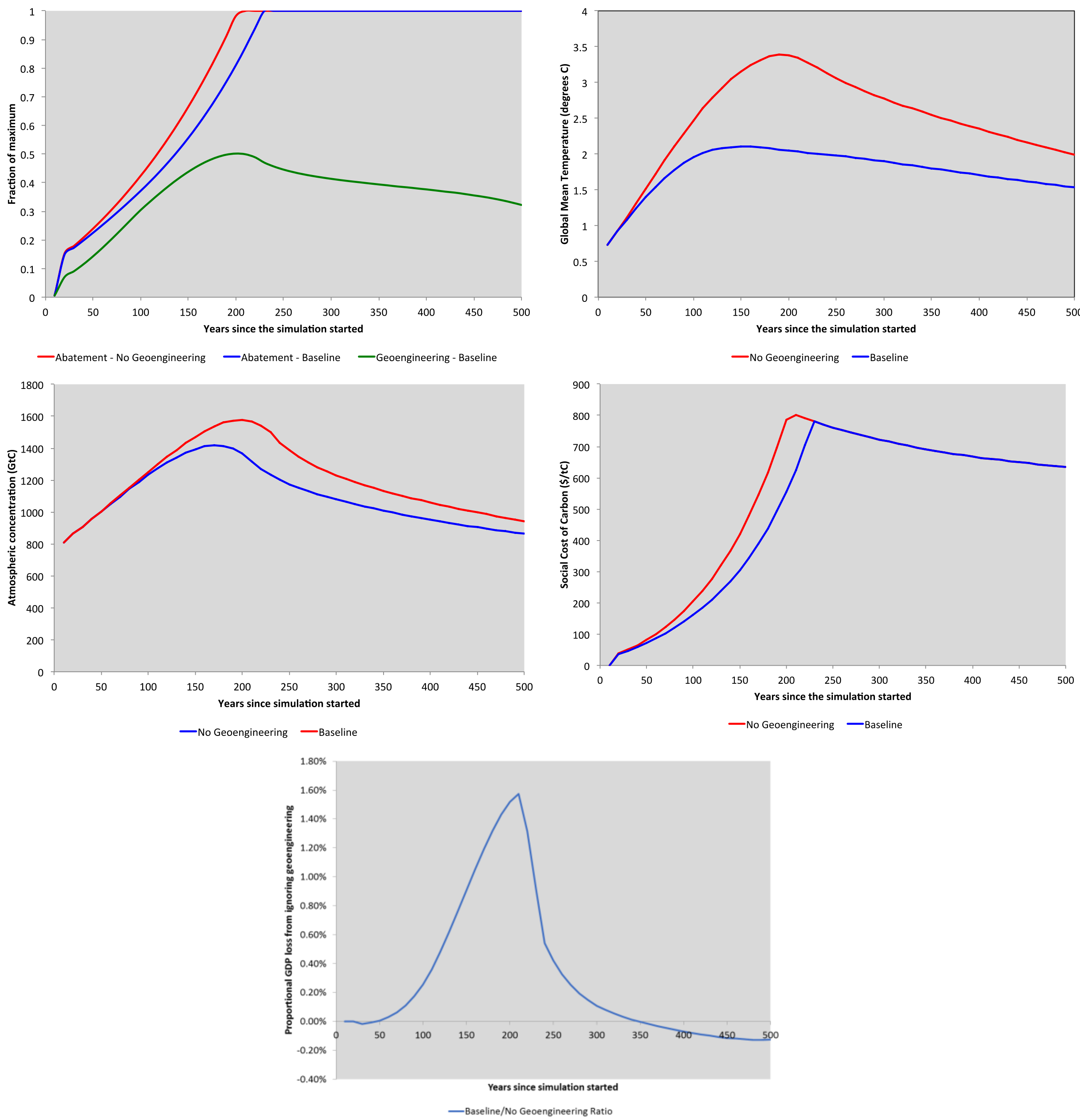


Figure 2 – Base case simulations

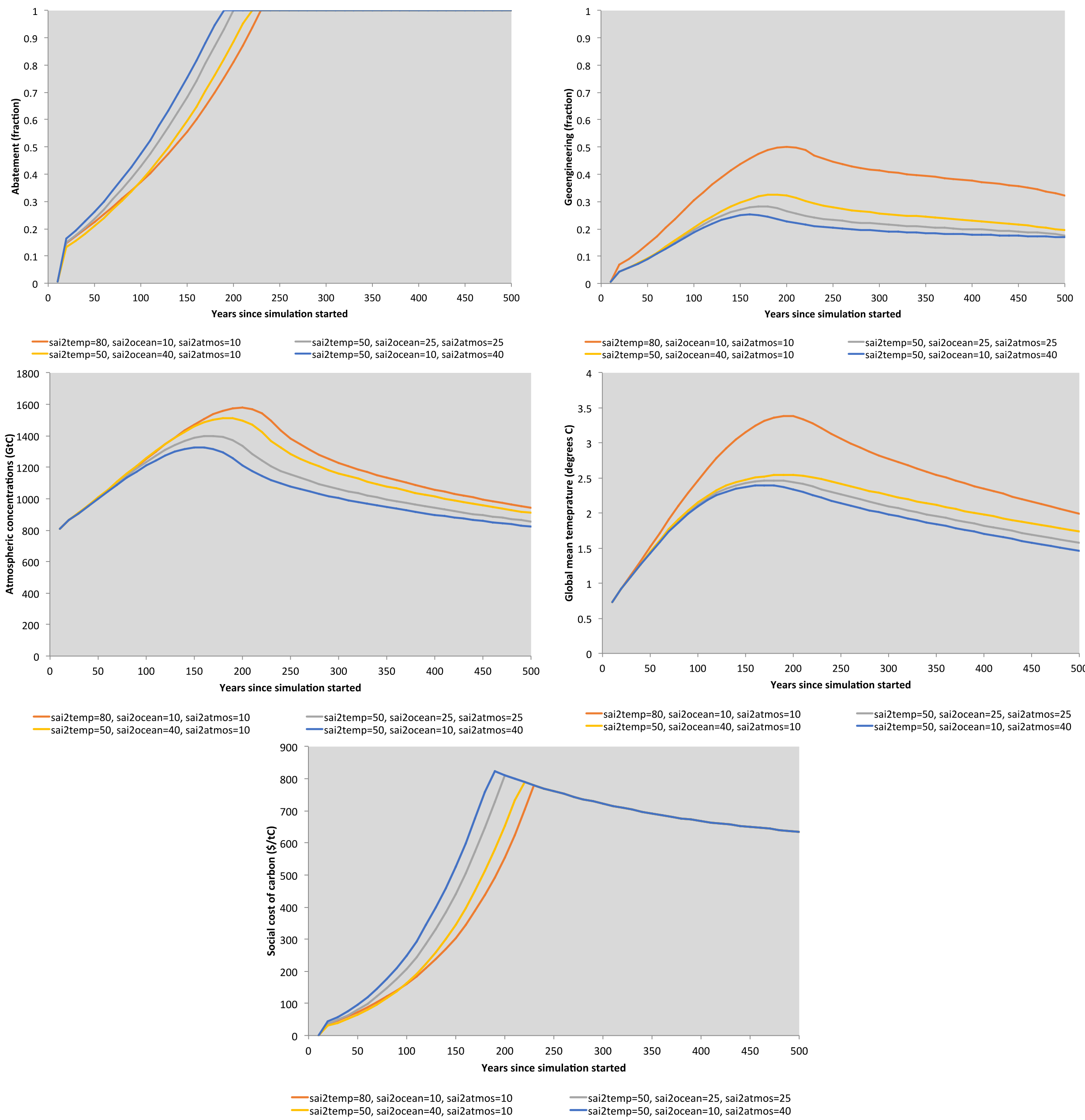


Figure 3 – Variation in the Composition of Damages

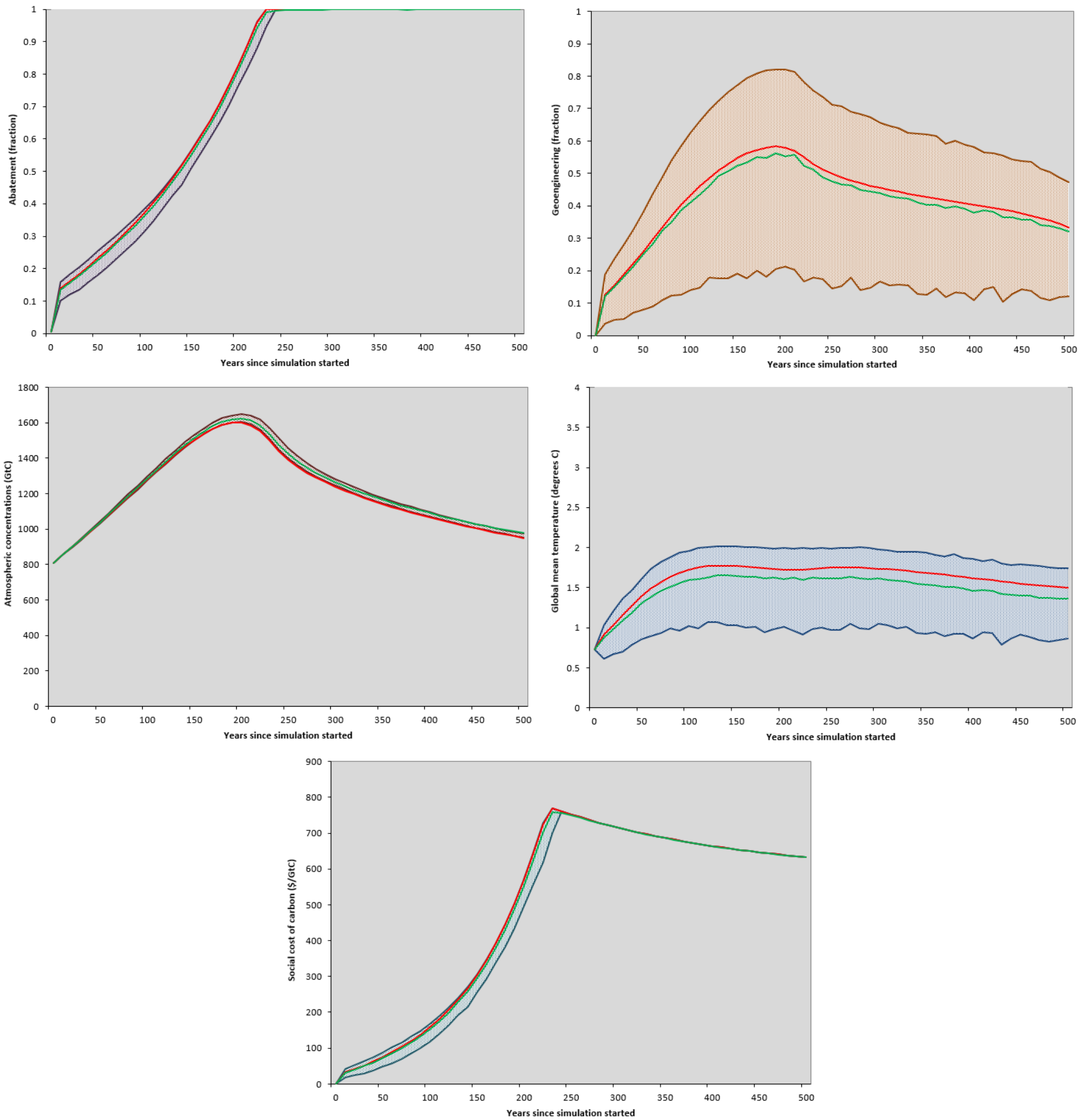


Figure 4 – Stochastic Simulation Results–Climate Sensitivity

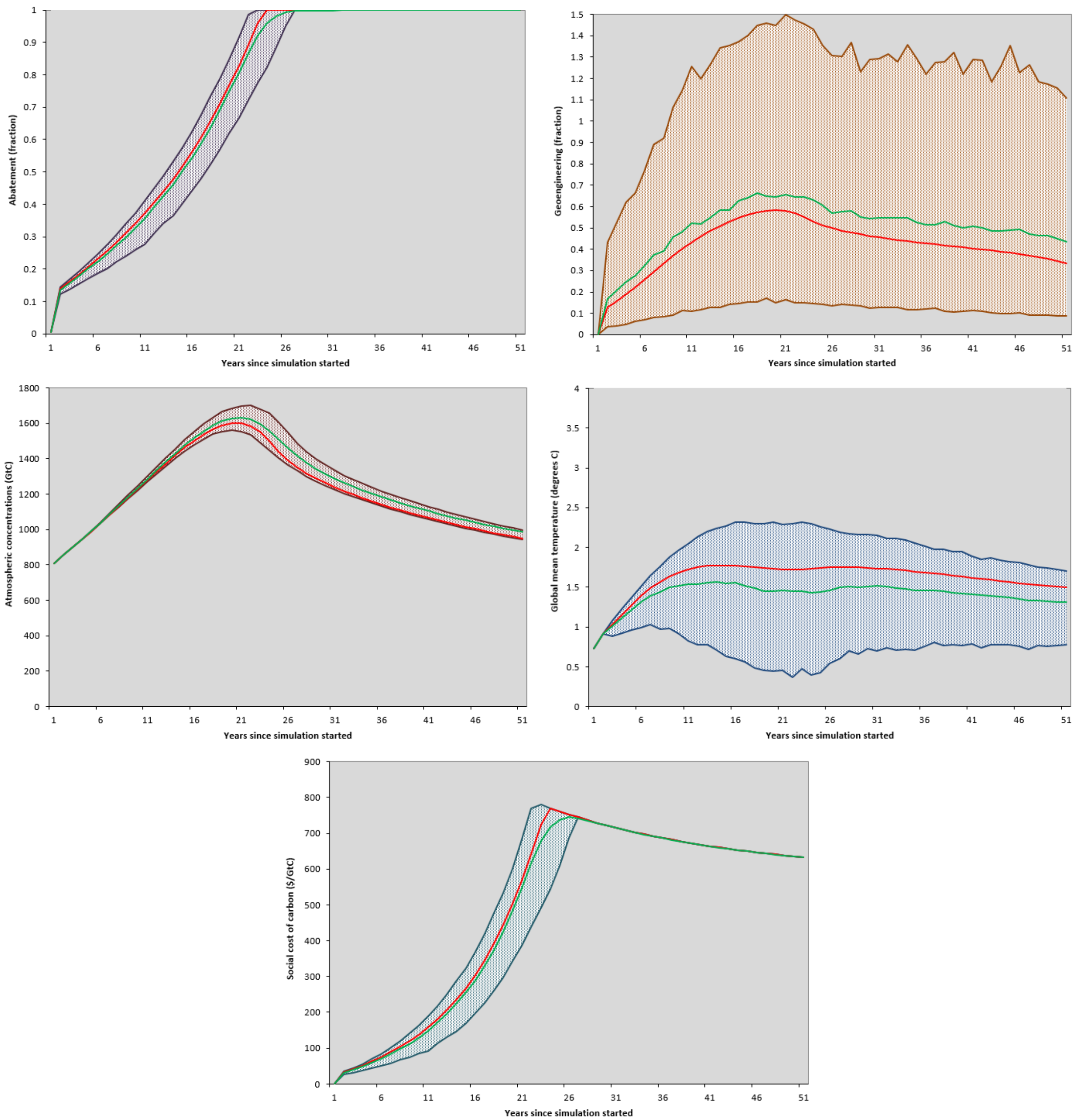


Figure 5 – Stochastic Simulation Results–Geoengineering damage

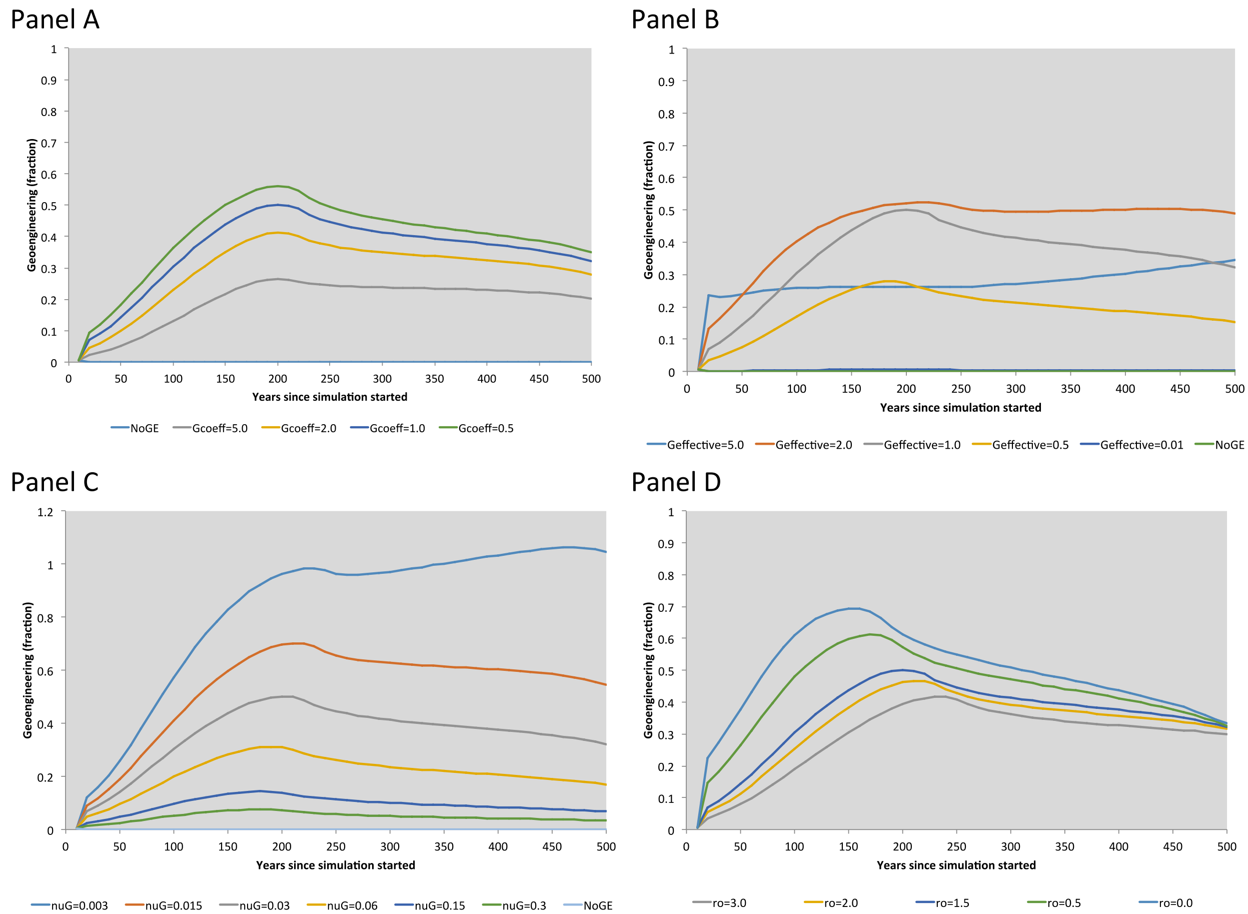


Figure 6 – Sensitivity analysis

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Appendix: Details of Static Model

In the appendix we verify that 1) the marginal cost of abatement is increasing in k_a, 2) the marginal benefit of abatement is decreasing in k_a, 3) the marginal cost of abatement is higher for a positive value of GE k_g than it is for $k_g = 0$, and 4) the marginal benefit of abatement is lower for a positive value of GE k_g than it is for $k_g = 0$. We also argue that the difference in marginal costs (3) is likely to be smaller in magnitude than the difference in marginal benefits (4).

The marginal cost of abatement as a function of abatement k_a is $f'(k - k_a - k_g)\left(1 - d\left(\left(1 - g(k_a)\right)k, k_g\right)\right)$, for some fixed k_g. The first half is increasing in k_a since f is concave. The second half is increasing in k_a since g is increasing and d is decreasing in x. Thus, marginal cost is monotone increasing.

The marginal benefit of abatement is $f(k - k_a - k_g)kg'(k_a)d_x\left(\left(1 - g(k_a)\right)k, k_g\right)$. Because f is increasing the first part is decreasing in k_a. Assuming that g is concave, the middle part is decreasing in k_a. Lastly, because the cross-partial derivative $d_{xk} < 0$, the third part of this expression is decreasing in k_a, and so the entire expression for marginal benefit is monotone decreasing.

The marginal cost of abatement at zero GE is $f'(k - k_a)\left(1 - d\left(\left(1 - g(k_a)\right)k, 0\right)\right)$, and for an arbitrary level of GE (for instance, k_g^{opt}) it is $f'(k - k_a - k_g)\left(1 - d\left(\left(1 - g(k_a)\right)k, k_g\right)\right)$. Because f is concave, the first part of the expression is higher for $k_g > 0$. Because $d_k < 0$, the second part of the expression is higher for $k_g > 0$. Thus, the marginal cost of abatement is higher for a positive value of GE k_g than it is for $k_g = 0$.

The marginal benefit of abatement at zero GE is $f(k - k_a)kg'(k_a)d_x\left(\left(1 - g(k_a)\right)k, 0\right)$, and for an arbitrary level of GE it is $f(k - k_a - k_g)kg'(k_a)d_x\left(\left(1 - g(k_a)\right)k, k_g\right)$. Because f is increasing, the first part of this expression is lower for $k_g > 0$. Because $d_{xk} < 0$, the second part of the expression is lower for $k_g > 0$. Thus, the marginal benefit of abatement is lower for a positive value of GE k_g than it is for $k_g = 0$.

Lastly, we argue that the magnitude of the difference in the two marginal benefit curves is likely to be large, while the magnitude of the difference in the two marginal cost curves is likely to be small. This cannot be mathematically demonstrated like the rest of the claims in the appendix. Rather, it follows from our intuition of the application of the model. Consider first the difference in marginal costs. The first half of the expression is the difference between $f'(k - k_a)$ and $f'\left(k - k_a - k_g\right)$. This is likely to be small, because k_g is likely to be very small relative to k (i.e. only a small fraction of total capital will be spent on GE). The second difference between the two expressions is $1 - d\left(\left(1 - g(k_a)\right)k, 0\right)$ versus $1 - d\left(\left(1 - g(k_a)\right)k, k_g\right)$. This is also likely to be small because the damages from climate change as a proportion of total potential output (d) is likely to be only a few percentage points. Thus, even if the optimal level of k_g completely eliminated climate change damages ($d = 0$), the value of $1 - d$ would change only from, say, 98% to 100%.

Consider instead the differences in marginal damages. The first difference is the difference between $f(k - k_a)$ and $f\left(k - k_a - k_g\right)$, which from the argument in the previous paragraph is likely to be small since k_g is small relative to k. The other difference is the difference between $d_x\left(\left(1 - g(k_a)\right)k, 0\right)$ and $d_x\left(\left(1 - g(k_a)\right)k, k_g\right)$. This difference is likely to be large (first-order). Even though damages d may be small (a few percentage points), the difference in the marginal damages d_x may be large depending on the presence of GE. At the extreme, if k_g is sufficiently high to eliminate any damages from climate change, then $d_x\left(x, k_g\right)$ will be zero though $d_x\left(x, k_g\right)$ is positive.

<u>Uncertainty</u>

We now derive the expressions in section II.A in which pollution damages and geoengineering benefits are uncertain. The two first-order conditions for the planner's problem can be written as:

$$F \equiv f'(k-k_a-k_g)\cdot\left(1-E\left[d\left(x,k_g,\theta_x,\theta_g\right)\right]\right)-f(k-k_a-k_g)\cdot k\cdot g'(k_a)\cdot E\left[d_x\left(x,k_g,\theta_x,\theta_g\right)\right] = 0$$

$$G \equiv g'(k_a)\cdot k\cdot E\left[d_x\left(x,k_g,\theta_x,\theta_g\right)\right]+E\left[d_k\left(x,k_g,\theta_x,\theta_g\right)\right]=0$$

The variance of the shocks, $Var(\theta_x)$ and $Var(\theta_g)$, are treated as exogenous parameters that affect the expected values of the damage function and its partial derivatives, as defined in the text. Therefore, the implicit function theorem can be used to find the following derivatives:

$$\begin{pmatrix}\dfrac{\partial k_a}{\partial Var(\theta_x)}\\ \dfrac{\partial k_g}{\partial Var(\theta_x)}\end{pmatrix}=-\begin{pmatrix}\dfrac{\partial F}{\partial k_a} & \dfrac{\partial F}{\partial k_g}\\ \dfrac{\partial G}{\partial k_a} & \dfrac{\partial G}{\partial k_g}\end{pmatrix}^{-1}\begin{pmatrix}\dfrac{\partial F}{\partial Var(\theta_x)}\\ \dfrac{\partial G}{\partial Var(\theta_x)}\end{pmatrix}$$

$$\begin{pmatrix}\dfrac{\partial k_a}{\partial Var(\theta_g)}\\ \dfrac{\partial k_g}{\partial Var(\theta_g)}\end{pmatrix}=-\begin{pmatrix}\dfrac{\partial F}{\partial k_a} & \dfrac{\partial F}{\partial k_g}\\ \dfrac{\partial G}{\partial k_a} & \dfrac{\partial G}{\partial k_g}\end{pmatrix}^{-1}\begin{pmatrix}\dfrac{\partial F}{\partial Var(\theta_g)}\\ \dfrac{\partial G}{\partial Var(\theta_g)}\end{pmatrix}$$

The inverse of the 4-by-4 matrix in these expressions (the Jacobian matrix) is

$$\frac{1}{Det}\begin{pmatrix}\dfrac{\partial G}{\partial k_g} & -\dfrac{\partial F}{\partial k_g}\\ -\dfrac{\partial G}{\partial k_a} & \dfrac{\partial F}{\partial k_a}\end{pmatrix}$$

The determinant of the Jacobian Det is positive from the second-order condition of the planner's maximization problem. The elements of the Jacobian matrix are:

$$\frac{\partial F}{\partial k_a}=-f''(k_p)(1-E[d])+f'(k_p)\left(-\frac{\partial E[d]}{\partial k_a}\right)+f'(k_p)kg'(k_a)E[d_x]-f(k_p)kg''(k_a)E[d_x] - f(k_p)kg'(k_a)\frac{\partial E[d_x]}{\partial k_a}>0$$

$$\frac{\partial F}{\partial k_g}=-f''(k_p)(1-E[d])+f'(k_p)\left(-\frac{\partial E[d]}{\partial k_g}\right)+f'(k_p)kg'(k_a)E[d_x]-f(k_p)kg'(k_a)\frac{\partial E[d_x]}{\partial k_g}>0$$

$$\frac{\partial G}{\partial k_a} = g''(k_a)kE[d_x] + g'(k_a)k\frac{\partial E[d_x]}{\partial k_a} + \frac{\partial E[d_k]}{\partial k_a}$$

$$\frac{\partial G}{\partial k_g} = g'(k_a)k\frac{\partial E[d_x]}{\partial k_g} + \frac{\partial E[d_k]}{\partial k_g}$$

All terms in $\frac{\partial F}{\partial k_a}$ and $\frac{\partial F}{\partial k_g}$ are positive. However, $\frac{\partial G}{\partial k_a}$ and $\frac{\partial G}{\partial k_g}$ have ambiguous sign. In $\frac{\partial G}{\partial k_a}$, the first two terms are negative, and the third term is positive. In $\frac{\partial G}{\partial k_g}$, the first term is negative, and the second term is positive. Since the final term in each expression is a multiple of d_{kk}, which we assume is negative, $\frac{\partial G}{\partial k_a}$ is negative and $\frac{\partial G}{\partial k_g}$ is positive so long as d_{xk} is not too negative.

Furthermore,

$$\frac{\partial F}{\partial Var(\theta_x)} = -f'(k_p)\frac{\partial E[d]}{\partial Var(\theta_x)} - f(k_p)kg'(k_a)\frac{\partial E[d_x]}{\partial Var(\theta_x)}$$

$$\frac{\partial G}{\partial Var(\theta_x)} = g'(k_a)k\frac{\partial E[d_x]}{\partial Var(\theta_x)} + \frac{\partial E[d_k]}{\partial Var(\theta_x)}$$

Substituting these expressions into the matrix equation above, simplifying, and collecting terms yields

$$\begin{aligned}\frac{\partial k_a}{\partial Var(\theta_x)} = \frac{1}{Det}\Bigg\{&\left[f'(k_p)\left(g'(k_a)k\frac{\partial E[d_x]}{\partial k_g} + \frac{\partial E[d_k]}{\partial k_g}\right)\right]\frac{\partial E[d]}{\partial Var(\theta_x)} \\ &+ \Bigg[f(k_p)kg'(k_a)\frac{\partial E[d_k]}{\partial k_g} \\ &+ g'(k_a)k\left(-f''(k_p)(1-E[d]) - f'(k_p)\frac{\partial E[d]}{\partial k_g} + f'(k_p)kg'(k_a)E[d_x]\right)\Bigg]\frac{\partial E[d_x]}{\partial Var(\theta_x)} \\ &+ \left[\frac{\partial F}{\partial k_g}\right]\frac{\partial E[d_k]}{\partial Var(\theta_x)}\Bigg\}\end{aligned}$$

Define $A \equiv f(k_p)kg'(k_a)\frac{\partial E[d_k]}{\partial k_g} + g'(k_a)k\left(-f''(k_p)(1-E[d]) - f'(k_p)\frac{\partial E[d]}{\partial k_g} + f'(k_p)kg'(k_a)E[d_x]\right) > 0$ and $B \equiv \frac{\partial F}{\partial k_g} > 0$, and the expression is as appears in the text.

Next,

$$\frac{\partial k_g}{\partial Var(\theta_x)} = \frac{1}{Det}\left\{\left[f'(k_p)\left(-g''(k_a)kE[d_x] - g'(k_a)k\frac{\partial E[d_x]}{\partial k_a} - \frac{\partial E[d_k]}{\partial k_a}\right)\right]\frac{\partial E[d]}{\partial Var(\theta_x)}\right.$$

$$+\left[-f'(k_p)kg'(k_a)\left(\frac{\partial E[d_k]}{\partial k_a}\right)\right.$$

$$\left. - g'(k_a)\left(-f''(k_p)(1-E[d]) - f'(k_p)\frac{\partial E[d]}{\partial k_a} + f'(k_p)kg'(k_a)E[d_x]\right)\right]\frac{\partial E[d_x]}{\partial Var(\theta_x)}$$

$$\left. + \left[-\frac{\partial F}{\partial k_a}\right]\frac{\partial E[d_k]}{\partial Var(\theta_x)}\right\}$$

Define $C \equiv f'(k_p)kg'(k_a)\left(\frac{\partial E[d_k]}{\partial k_a}\right) + g'(k_a)\left(-f''(k_p)(1-E[d]) - f'(k_p)\frac{\partial E[d]}{\partial k_a} + f'(k_p)kg'(k_a)E[d_x]\right) > 0$ and $D \equiv \frac{\partial F}{\partial k_a} > 0$, and the expression is as appears in the text.

The solutions for $\frac{\partial k_a}{\partial Var(\theta_g)}$ and $\frac{\partial k_g}{\partial Var(\theta_g)}$ are identical to those for $\frac{\partial k_a}{\partial Var(\theta_x)}$ and $\frac{\partial k_g}{\partial Var(\theta_x)}$, respectively, except for replacing all partials with respect to $Var(\theta_x)$ with partials with respect to $Var(\theta_g)$.

<u>Decomposition of Climate Damages</u>

We now derive the expressions in section II.B where damages occur from both temperature and from carbon. The first-order conditions in for the planner's problem are identical as in the original model, except that the damage function is now $d(x;k_g) = \lambda_d d_1(x) + d_2(x;k_g)$.

$$F \equiv f'(k-k_a-k_g)\cdot\left(1-d(x;k_g)\right) - f(k-k_a-k_g)\cdot k\cdot g'(k_a)\cdot d_x(x;k_g) = 0$$

$$G \equiv g'(k_a)\cdot k\cdot d_x(x;k_g) + d_k(x;k_g) = 0$$

As with the last model, the implicit function theorem can be used to conduct comparative statics:

$$\begin{pmatrix}\frac{\partial k_a}{\partial \lambda_d}\\ \frac{\partial k_g}{\partial \lambda_d}\end{pmatrix} = -\begin{pmatrix}\frac{\partial F}{\partial k_a} & \frac{\partial F}{\partial k_g}\\ \frac{\partial G}{\partial k_a} & \frac{\partial G}{\partial k_g}\end{pmatrix}^{-1}\begin{pmatrix}\frac{\partial F}{\partial \lambda_d}\\ \frac{\partial G}{\partial \lambda_d}\end{pmatrix}$$

Again, it can be shown that $\frac{\partial F}{\partial k_a} > 0$, $\frac{\partial F}{\partial k_g} > 0$, and the determinant of the Jacobian is positive. The partial derivatives $\frac{\partial G}{\partial k_a}$ and $\frac{\partial G}{\partial k_g}$ have ambiguous sign:

$$\frac{\partial G}{\partial k_a} = g''(k_a)kd_{2x} + g'(k_a)k\frac{\partial Ed_{2x}}{\partial k_a} + \frac{\partial Ed_{2k}}{\partial k_a}$$

$$\frac{\partial G}{\partial k_g} = g'(k_a)k\frac{\partial Ed_{2x}}{\partial k_g} + \frac{\partial Ed_{2k}}{\partial k_g}$$

As in the model in the prior section, the first two terms in $\frac{\partial G}{\partial k_a}$ are negative, and the last is positive. The first term in $\frac{\partial G}{\partial k_g}$ is negative, and the last is positive. Also,

$$\frac{\partial F}{\partial \lambda_d} = -f'\left(k_p\right)d_1 - f\left(k_p\right)d_{1x}kg'(k_a) < 0$$

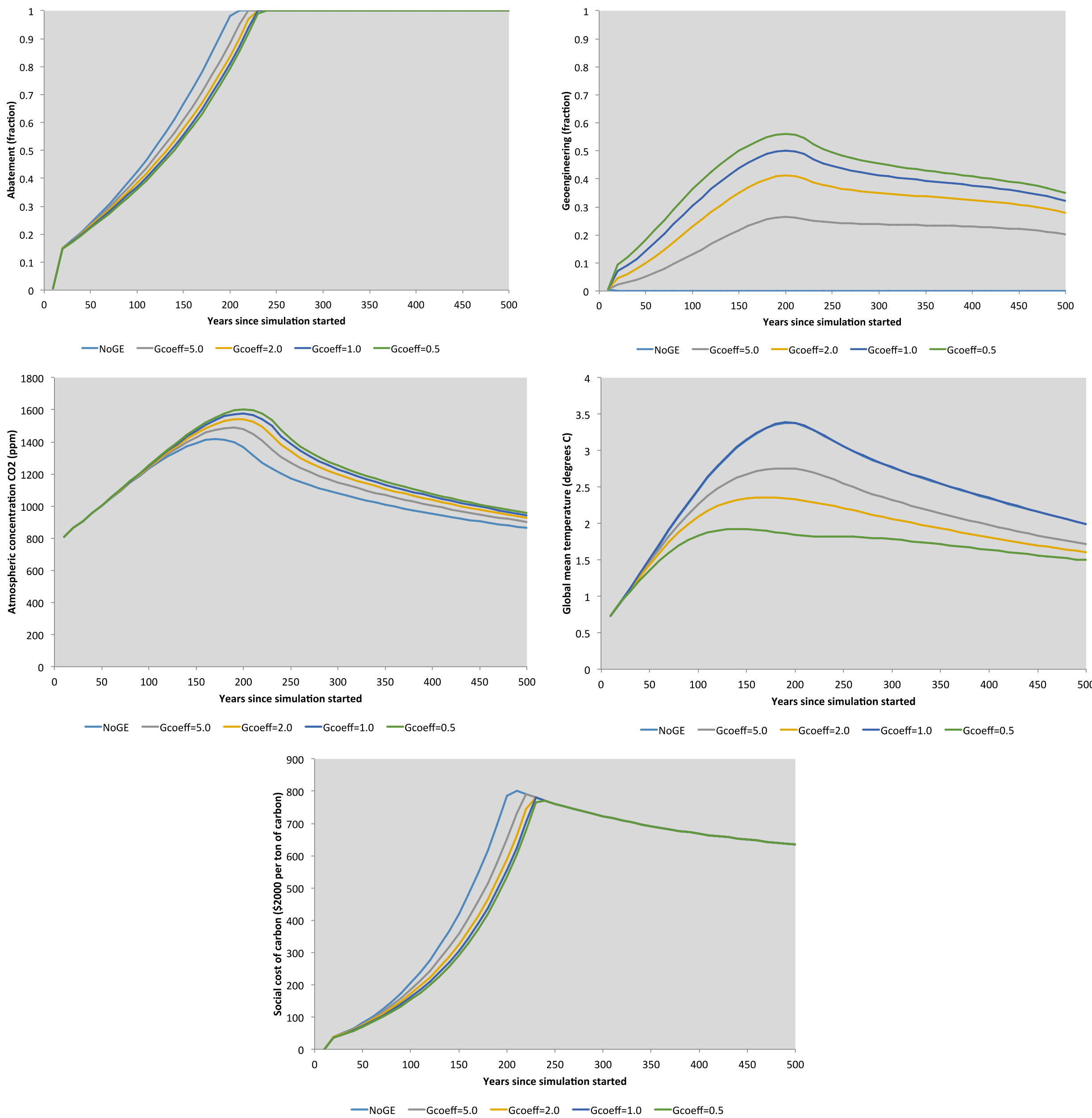
$$\frac{\partial G}{\partial \lambda_d} = 0$$

After substituting in for each of these partial derivatives and simplifying, we get

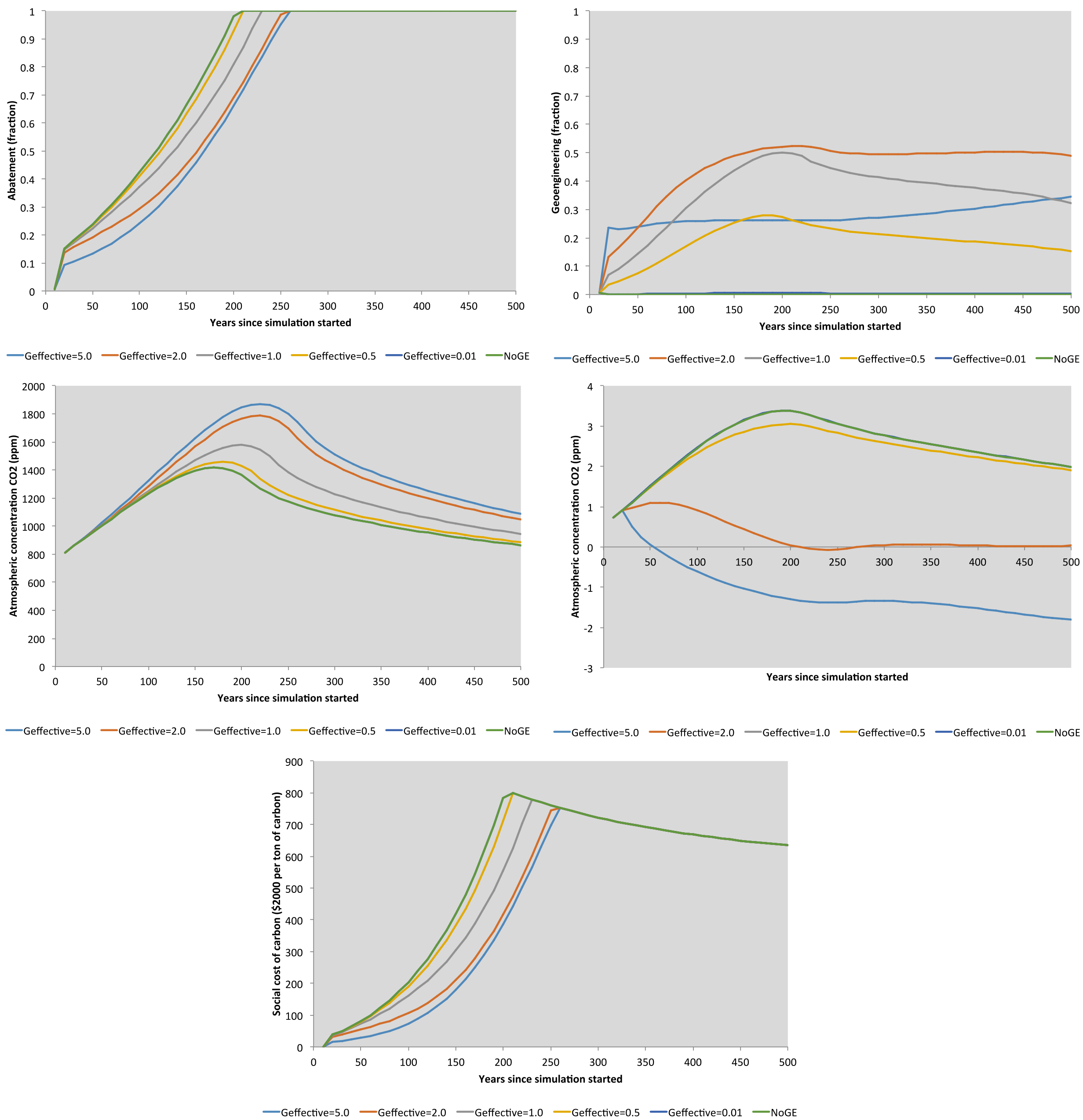
$$\frac{\partial k_a}{\partial \lambda_d} = \frac{1}{Det}\cdot\left[f'\left(k_p\right)d_1 + f\left(k_p\right)d_{1x}kg'(k_a)\right][g'(k_a)kd_{2xk} + d_{2kk}]$$

$$\frac{\partial k_g}{\partial \lambda_d} = \frac{1}{Det}\cdot\left[f'\left(k_p\right)d_1 + f\left(k_p\right)d_{1x}kg'(k_a)\right][(-g''(k_a)kd_{2x} + g'(k_a)^2k^2d_{2xx} - kg'(k_a)d_{2xk})]$$

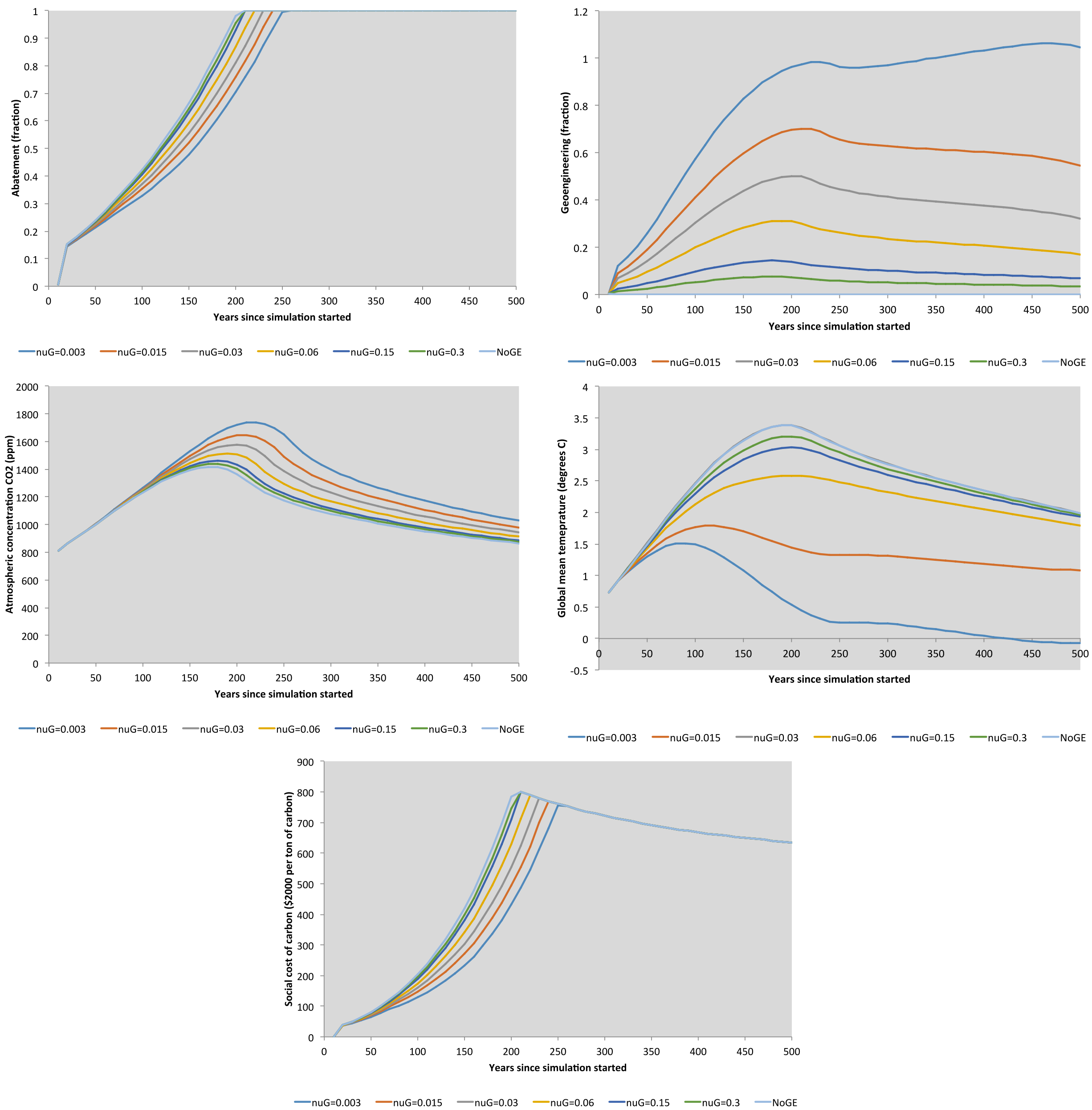
The first set of terms in front of each expression, $\frac{1}{Det}\cdot\left[f'\left(k_p\right)d_1 + f\left(k_p\right)d_{1x}kg'(k_a)\right]$, is positive and is defined as the constant C in the text.



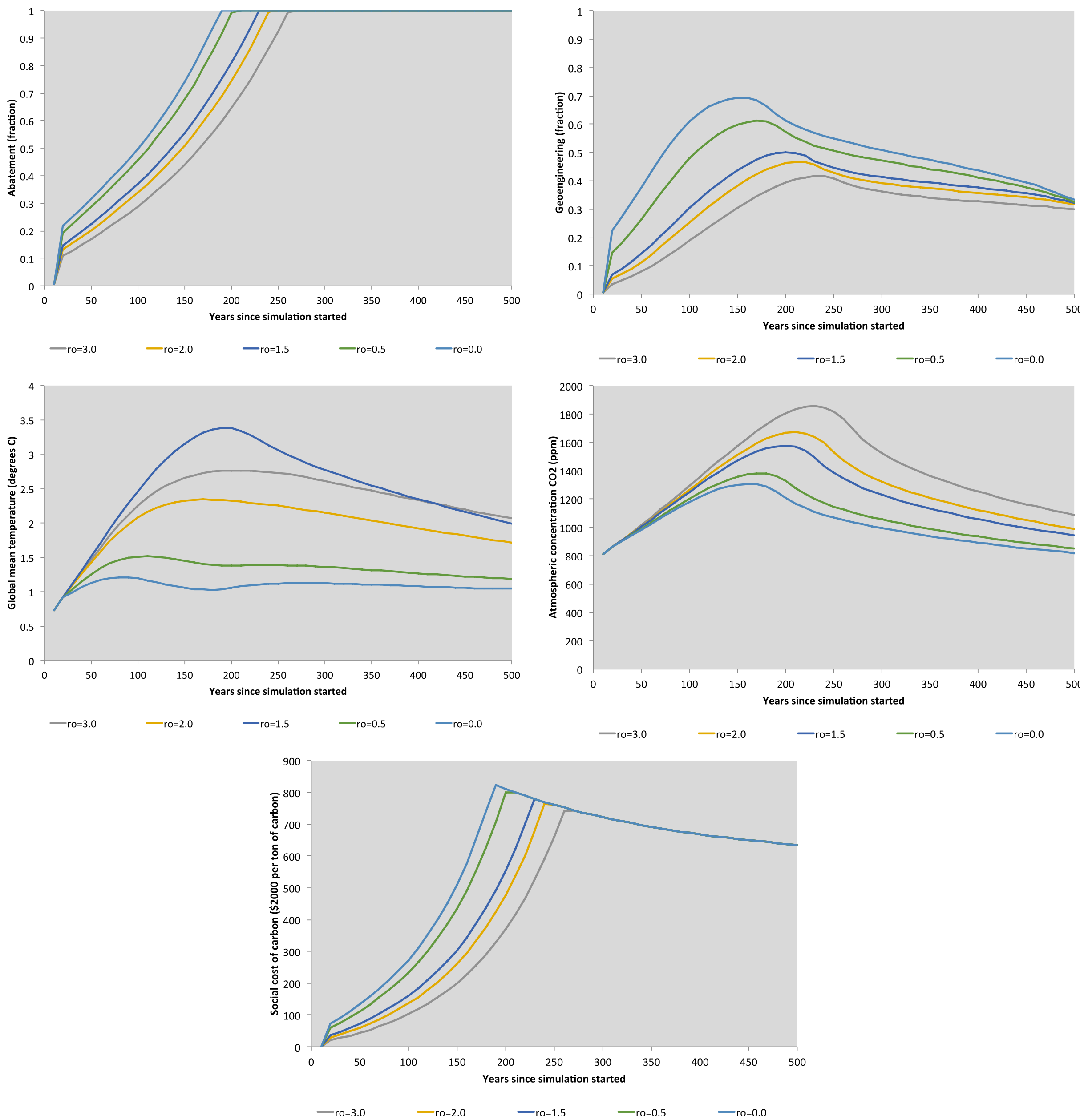
Appendix Figure 1 – Sensitivity analysis – Cost of geoengineering



Appendix Figure 2 – Sensitivity analysis – Effectiveness of geoengineering



Appendix Figure 3 – Sensitivity analysis – Damages from geoengineering



Appendix Figure 4 – Sensitivity analysis – Discount Rate